Problem 1 (10 points). Evaluate the following limits. JUSTIFY your answers. If a limit does not exist, say so.

(1) $\lim_{n\to\infty} \frac{2+3n^2}{3+2n^2}$

(2) $\lim_{n\to\infty} \frac{(\ln n)^2}{n}$

(3) $\lim_{n\to\infty} \frac{\sin(n)+n}{2n}$

(4) $\lim_{n\to\infty} (-1)^n e^{-n}$

(5) $\lim_{n\to\infty} \sin\left(\frac{\pi n+3}{2n}\right)$

Problem 2 (10 points). Determine whether the following series converge or diverge. JUSTIFY your answers. You do NOT need to find the sum.

(1) $\sum_{n=0}^{\infty} \frac{3n+2}{2n^3-n+1}$

(2) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(3) $\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$

(4) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

(5) $\sum_{n=1}^{\infty} \frac{2^n n^2}{n!}$

Problem 3 (6 points). Find the Maclaurin series (Taylor series at zero) of the following functions. You do NOT need to find the radius of convergence.

(1) $f(x) = \begin{cases} \frac{1-\cos(x)}{x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

(2) $f(x) = \sqrt{4 - x^2}$.

(3) $f(x) = \int_0^x ze^{z^2}\, dz$

Problem 4 (6 points). Consider the power series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{5^n n^2}.$$

(1) Where is this power series centered?

(2) What is its radius of convergence?

(3) What is its interval of convergence?

Problem 5 (10 points). For each statement below, circle T or F according to whether the statement is true or false. You do NOT need to justify your answers.

T F If the series $\sum_{n=0}^{\infty} |a_n|$ converges, then the series $\sum_{n=0}^{\infty} a_n$ must also converge.

T F If the series $\sum_{n=0}^{\infty} b_n$ converges and $0 \le a_n \le b_n$, then the series $\sum_{n=0}^{\infty} a_n$ must also converge.

T F The series $\sum_{n=1}^{\infty}(-4/3)^n$ converges.

T F The series $\sum_{n=1}^{\infty}(\sqrt{n+1} - \sqrt{n})$ diverges.

T F The series $\sum_{n=1}^{\infty}(1 - 1/n^2)$ diverges.

T F If the series $\sum_{n=0}^{\infty} a_n$ converges to S, then the series $\sum_{n=1}^{\infty} a_n$ converges to $S - a_0$.

T F If the series $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ both converge, then $\lim_{n\to\infty} a_n/b_n = L$ for some nonzero real number L.

T F If the function f is continuous, positive, and decreasing on $[1, \infty)$ and $\int_1^{\infty} f(x)\, dx$ converges, then $\sum_{n=1}^{\infty} f(n)$ converges to the same value.

T F If the power series $\sum_{n=1}^{\infty} c_n x^n$ converges for $x = 1$ then it must also converge for $x = -1$.

T F If the series $\sum_{n=0}^{\infty} a_n$ converges, then the sequence of terms $\{a_n\}_{n=0}^{\infty}$ must converge to zero.

Problem 6 (10 points). For each statement below, circle T or F according to whether the statement is true or false. You do NOT need to justify your answers.

T F The series $\sum_{n=1}^{\infty}(-1)^n/n^2$ converges conditionally.

T F If the sequence $\{b_n\}_{n=0}^{\infty}$ is bounded and increasing, then it must converge.

T F If the series $\sum_{n=0}^{\infty} a_n$ diverges and $0 \leq a_n \leq b_n$ for all n, then the series $\sum_{n=0}^{\infty} b_n$ must also diverge.

T F If the series $\sum_{n=0}^{\infty} a_n$ converges to L, then the series $\sum_{n=0}^{\infty}(a_n + 3)$ converges to $L + 3$.

T F The series $\sum_{n=1}^{\infty}\left(\frac{1}{n+2} - \frac{1}{n}\right)$ converges.

T F Given a sequence $\{a_n\}_{n=0}^{\infty}$, if the sequences $\{a_{2n}\}_{n=0}^{\infty}$ and $\{a_{2n+1}\}_{n=0}^{\infty}$ both converge then $\{a_n\}_{n=0}^{\infty}$ itself must converge.

T F If the series $\sum_{n=0}^{\infty} b_n$ converges and $a_n/b_n \to 0$ as $n \to \infty$, then the series $\sum_{n=0}^{\infty} a_n$ must also converge.

T F The series $\sum_{n=1}^{\infty} n^{-1.1}$ converges.

T F If the power series $\sum_{n=0}^{\infty} c_n x^n$ converges absolutely at $x = -2$ then it must also converge absolutely at $x = 2$.

T F The series $\sum_{n=1}^{\infty} 1/3^{n-1}$ converges to $4/3$.

If you finish early, check your work on each problem by one of the following methods:
(1) Check that your proposed solution has the desired properties.
(2) Solve the problem again in a different way.
(3) For each true/false problem you marked as true, try to prove it is true.
(4) For each true/false problem you marked as false, try to construct a counterexample.