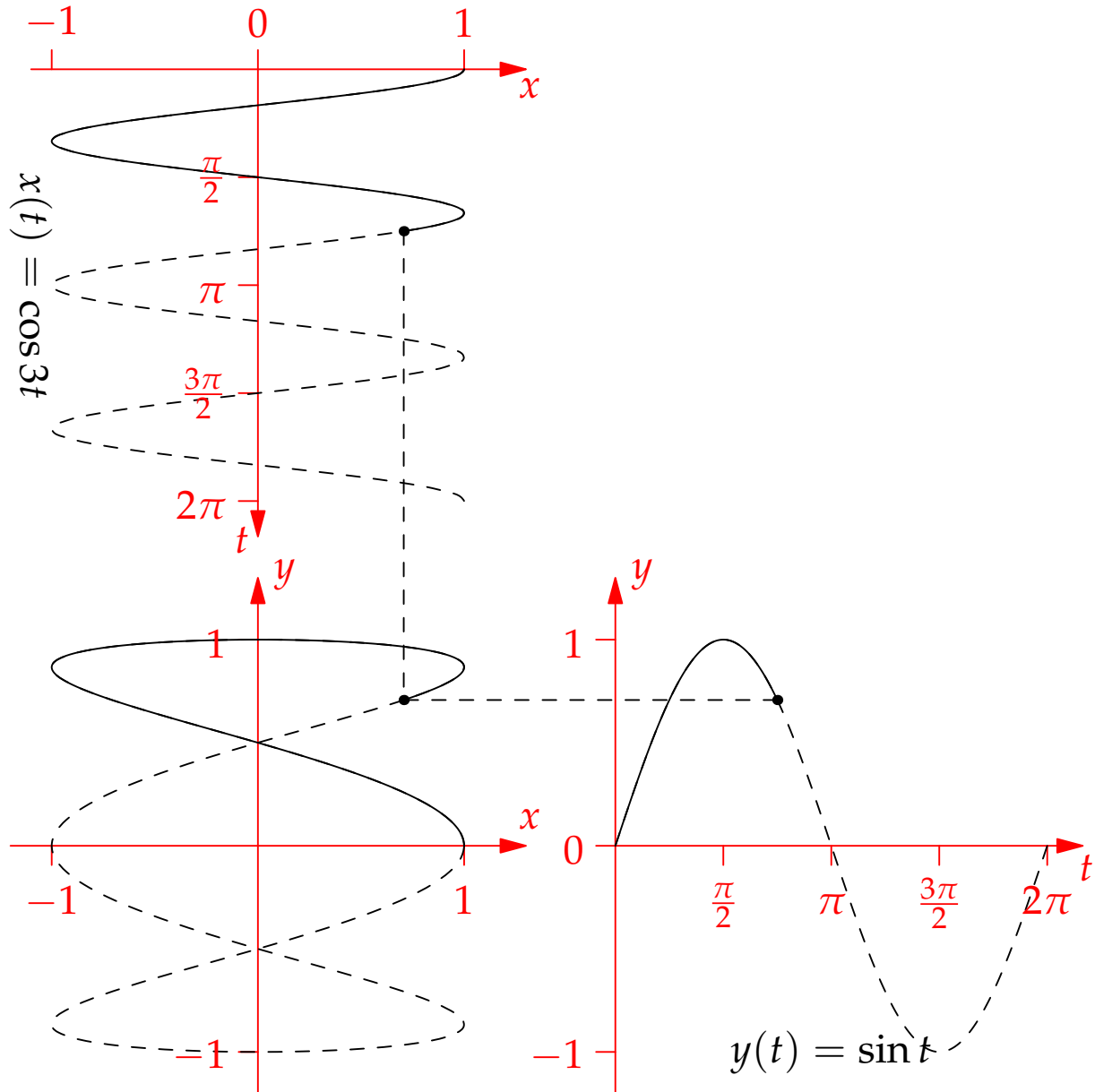


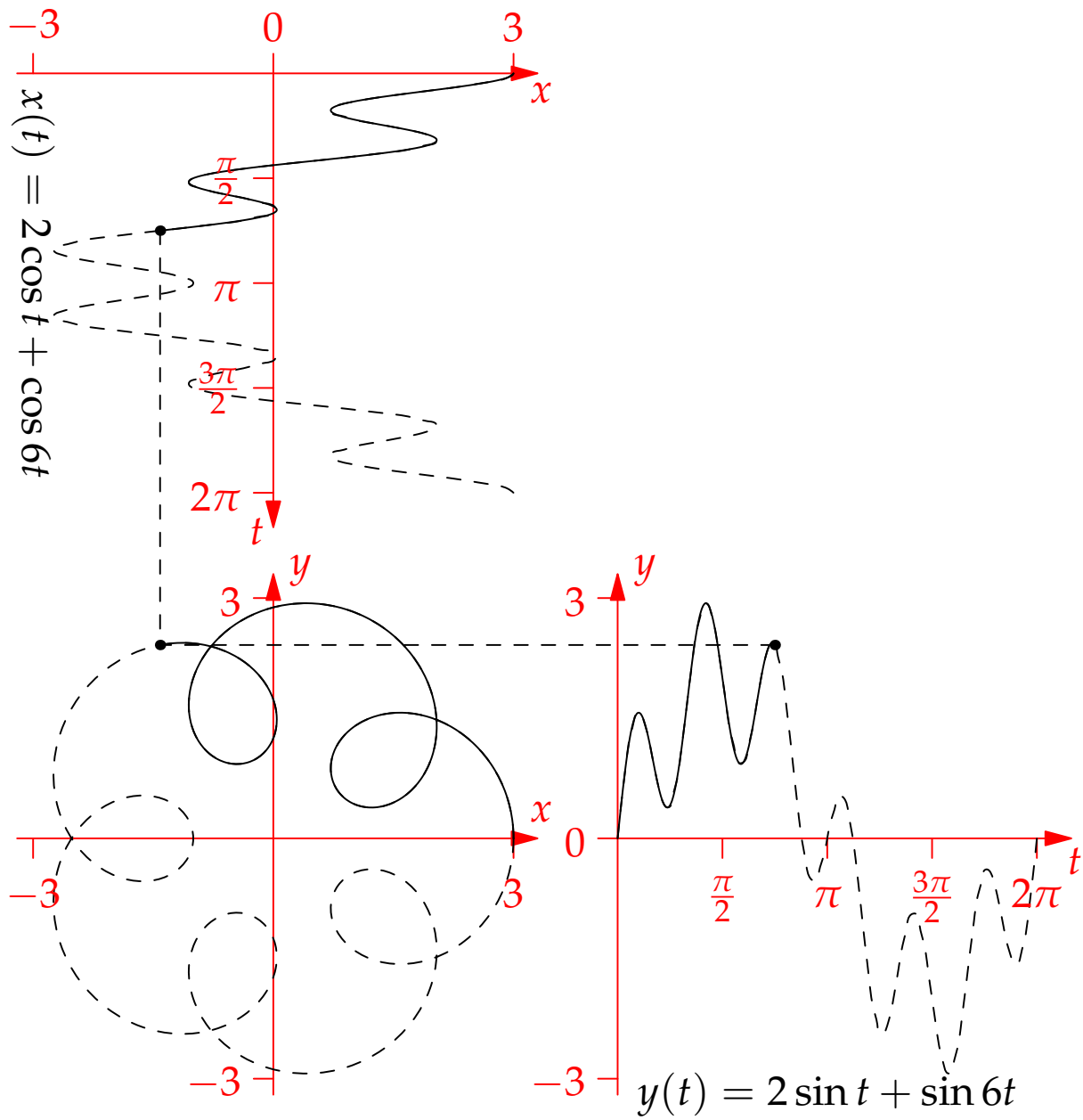
# Hourglass Animation

$$(x(t), y(t)) = (\cos 3t, \sin t), \quad 0 \leq t < 2\pi$$



## Five-petals Animation

$$(x(t), y(t)) = (2 \cos t + \cos 6t, 2 \sin t + \sin 6t), 0 \leq t < 2\pi$$



## Cycloid Animation

Circle radius  $a$  rolls along  $x$  axis

At time  $t$ , moved distance  $at$  to the right

Center of circle  $C = (at, a)$  at time  $t$

Want co-ordinates of  $P$

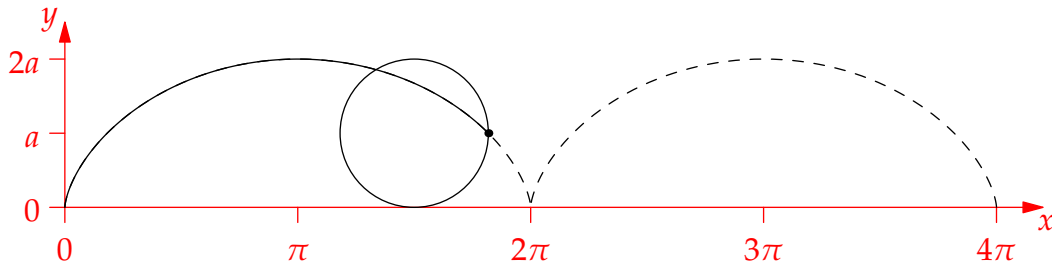
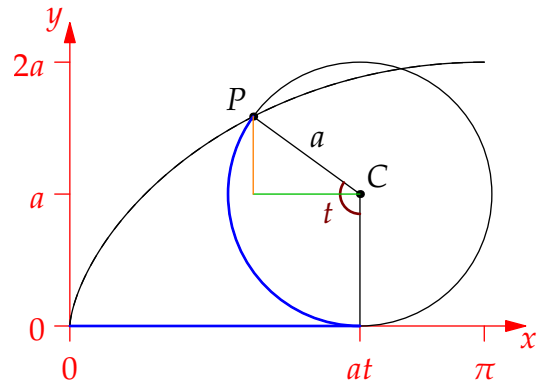
Arc-length equals horizontal distance  $at$

Angle (in radians) equals  $t$

Use right-triangle to get from  $C$  to  $P$ :

$$x = at - a \cos \left( t - \frac{\pi}{2} \right) = at - a \sin t$$

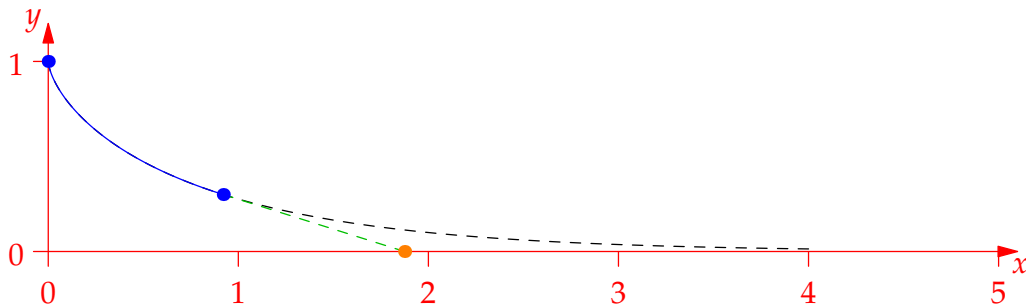
$$y = a + a \sin \left( t - \frac{\pi}{2} \right) = a - a \cos t$$



## Tractrix Animation

$$(x(t), y(t)) = \left( t - \frac{e^t - e^{-t}}{e^t + e^{-t}}, \frac{2}{e^t + e^{-t}} \right), t \geq 0: \quad \frac{dy}{dx} = \frac{-2}{e^t - e^{-t}}$$

- Distance between  $(x(t), y(t))$  and  $(t, 0)$  always = 1.
- Tangent line to curve joins these points.
- Describes path of a weight/dog starting at  $(0, 1)$  being pulled/dragged on a leash by a person walking at speed 1 along the  $x$ -axis.



## Cardioid Animation

$$r(\theta) = 2a(1 - \sin \theta), \quad 0 \leq t < 2\pi$$

( $\theta$  labeled mainly in **degrees** for clarity)

