Southern California Number Theory Day: **Abstracts**
UC Irvine, October 19, 2024

Micah Milinovich, University of Mississippi

Fourier optimization, prime gaps, and the least quadratic non-residue

There are many situations where one imposes certain conditions on a function and its Fourier transform and then attempts to optimize a certain quantity. I will describe how two such Fourier optimization frameworks can be used to study classical problems in number theory: bounding the maximum gap between consecutive primes assuming the Riemann hypothesis and bounding for the size of the least quadratic non-residue modulo a prime assuming the generalized Riemann hypothesis (GRH) for Dirichlet L-functions. The resulting extremal problems in analysis can be stated in accessible terms, but finding the exact answer appears to be rather subtle. However, we can experimentally find upper and lower bounds for our desired quantity that are numerically close. If time allows, I will discuss how a similar Fourier optimization framework can be used to bound the size of the least prime in an arithmetic progression on GRH. This is based upon joint works with E. Carneiro (ICTP), E. Quesada-Herrera (Lethbridge), A. Ramos (SISSA), and K. Soundararajan (Stanford).

Katherine Stange, University of Colorado

The local-global conjecture for Apollonian circle packings is false

Primitive integral Apollonian circle packings are fractal arrangements of tangent circles with integer curvatures. The curvatures form an orbit of a 'thin group,' a subgroup of an algebraic group having infinite index in its Zariski closure. The curvatures that appear must fall into one of six or eight residue classes modulo 24. The twenty-year-old local-global conjecture states that every sufficiently large integer in one of these residue classes will appear as a curvature in the packing. We prove that this conjecture is false for many packings, by proving that certain quadratic and quartic families are missed. The new obstructions are a property of the thin Apollonian group (and not its Zariski closure), and are a result of quadratic and quartic reciprocity, reminiscent of a Brauer-Manin obstruction. Based on computational evidence, we formulate a new conjecture. This is joint work with Summer Haag, Clyde Kertzer, and James Rickards. Time permitting, I will discuss some new results, joint with Rickards, that extend these phenomena to certain settings in the study of continued fractions.

Ziyang Gao, UCLA

Generic positivity of the Beilinson-Bloch height of Gross-Schoen and Ceresa cycles

Given an algebraic curve defined over a number field, one can define the Néron-Tate height on the Jacobian and prove its positivity. This height pairing and its positivity play important roles in the proof of the Mordell-Weil theorem, in Vojta's proof of the Mordell conjecture, and in the formulation of the BSD conjecture. The Jacobian can be seen, via the Abel-Jacobi map, as the moduli space of 0-cycles of degree 0 on the algebraic curve. The analogue for higher cycles was studied by Weil, Griffiths, Beilinson, and Bloch. In particular in the 1980s, Beilinson and Bloch independently proposed a conditional definition of heights for arbitrary homologically trivial cycle. The positivity of their heights, as conjectured by Beilinson and Bloch, is widely open.

In this talk, I will report a recent joint work with Shouwu Zhang about a generic positivity for the Gross-Schoen and Ceresa cycles of curves of genus at least 3. These are the simplest situation where the Beilinson-Bloch heights are unconditionally defined.

Alexandre de Faveri, Stanford University

Non-vanishing for cubic Hecke L-functions

We prove that a positive proportion of Hecke L-functions associated to the cubic residue symbol modulo squarefree Eisenstein integers do not vanish at the central point. Our principal new contribution is the asymptotic evaluation of the mollified second moment with power saving error term. No asymptotic formula for the mollified second moment of a cubic family was previously known (even over function fields). Our new approach makes crucial use of Patterson's evaluation of the Fourier coefficients of the cubic metaplectic theta function, Heath-Brown's cubic large sieve, and a Lindelöf-on-average upper bound for the second moment of cubic Dirichlet series that we establish. The significance of our result is that the (unitary) family considered does not satisfy a perfectly orthogonal large sieve bound. This is quite unlike other families of Dirichlet L-functions in the literature for which unconditional results are known: the symplectic family of quadratic characters and the unitary family of all Dirichlet characters modulo q. Consequently, our proof has fundamentally different features from the corresponding works of Soundararajan and of Iwaniec and Sarnak. Joint work with Chantal David, Alexander Dunn, and Joshua Stucky.

Lightning Talks

Meghan Lee, Wake Forest University

An Algorithm for Isolated Points on $X_0(n)$

An isolated point of degree d is a point on an algebraic curve which is not part of an infinite family of degree d points parametrized by some geometric object. We develop an algorithm to test whether a rational, non-CM j-invariant j gives rise to an isolated point on the modular curve $X_0(n)$, for any $n \in \mathbb{Z}^+$, using key results from Menendez and Zywina. This extends the prior algorithm of Bourdon et al. which tests whether a rational, non-CM j-invariant j gives rise to an isolated point on any modular curve $X_1(n)$. From this work, we determine that the set of j-invariants corresponding to isolated points on $X_1(n)$ is neither a subset nor a superset of those corresponding to isolated points on $X_0(n)$.

Chris Xu, UCSD

Equationless Chabauty for modular curves

For many high genus curves over $\mathbb{Q}$, the Chabauty-Coleman method provides a powerful method to completely determine its rational points, and hence should in theory be applicable to modular curves. However, a massive computational bottleneck comes in working with the equations defining a modular curve. In this talk, we describe an algorithm that bypasses this need. Time permitting, we will discuss computational aspects.

Tingyu Tao, UCI

The Eighth Moment of Dirichlet L-functions over Function Fields

We prove an asymptotic formula for the eighth moment of Dirichlet L-functions over function fields $\mathbb{F}_q(T)$ with an average over a fixed degree, which is the function field analogue of the result by Chandee and Li over the number field $\mathbb{Q}$. The result also confirms the existing random matrix theory predictions made by Andrade and Keating.

Mulun Yin, UCSB

Anticyclotomic Iwasawa theory for newforms at Eisenstein primes

Anticyclotomic Iwasawa theory for elliptic curves in the Eisenstein case was first studied in the recent work of Castella–Grossi–Lee–Skinner. We will talk about generalizations of their results to newforms of arbitrary weights, meanwhile removing their techinical assumptions. Using Hida arguments, one could also deduce weight 2 results for multiplicative reduction. The applications include new cases of p-part BSD formula, p-converse theorems and improvements in arithmetic statistics. This is joint work with Timo Keller.