

## Math 194, problem set #4

For discussion October 22

- (1) What is the remainder when the polynomial  $f(x)$  is divided by  $(x - a)^2$ ? by  $x^2 - a$ ?
- (2) For which real values of  $p$  and  $q$  are the roots of the polynomial  $x^3 - px^2 + 11x - q$  three successive integers? Give the roots in these cases.
- (3) Find, without a calculator, the smallest integer larger than  $(\sqrt{3} + \sqrt{2})^6$ .
- (4) (a) Determine all  $(a, b, c)$  such that the quadratic polynomial,  $Q(n) = an^2 + bn + c$ , assumes integer values for every integer  $n$ .  
(b) Determine all  $(a, b, c, d)$  such that the cubic polynomial,  $C(n) = an^3 + bn^2 + cn + d$ , assumes integer values for every integer  $n$ .
- (5) If  $P_n(x)$  denotes a polynomial of degree  $n$  such that  $P_n(k) = \frac{1}{k}$  for  $k = 1, 2, 3, \dots, n + 1$ , determine  $P_n(n + 2)$ .
- (6) For which real numbers  $c$  is there a straight line that intersects the curve
$$y = x^4 + 9x^3 + cx^2 + 9x + 4$$
in 4 distinct points? (Putnam 1994)
- (7) Let  $k$  be a positive integer. Find all polynomials  $P(x)$  with coefficients that are real numbers, satisfying the identity
$$P(P(x)) = P(x)^k.$$
- (8) For which ordered pairs of real numbers  $(b, c)$  do both of the roots of the quadratic polynomial  $z^2 + bz + c$  lie inside the unit disk  $\{z : |z| < 1\}$ ? (Putnam 1975)
- (9) Let  $f(x)$  be a polynomial, and  $a \neq b$ . Suppose  $f(x)$  leaves the remainder  $A$  when divided by  $x - a$  and the remainder  $B$  when divided by  $x - b$ . Find the remainder when  $f(x)$  is divided by  $(x - a)(x - b)$ .
- (10) Is there an infinite sequence of nonzero real numbers  $a_0, a_1, a_2, \dots$  such that the polynomial  $a_0 + a_1x + a_2x^2 + \dots + a_nx^n$  has exactly  $n$  real roots?