Math 194, problem set #3

For discussion October 15

(1) Prove that $36^{36} + 41^{41}$ is divisible by 77.

(2) A positive integer's digits are all 6 or 0; can it be be a perfect square?

(3) Show that $x^2 - y^2 = a^3$ always has positive integer solutions for x and y whenever a is an integer greater than one. For which values of a is the solution unique?

(4) What is the smallest natural number that leaves remainders $1, 2, 3, 4, 5, 6, 7, 8$ and 9 when divided by $2, 3, 4, 5, 6, 7, 8, 9$ and 10, respectively?

(5) Determine all n such that the n-digit number $R_n = 1111 \cdots 111$ is divisible by 37. For which n is it divisible by 41?

(6) Show that there is a sequence of 10^6 consecutive positive integers, each of which is divisible by the cube of some integer greater than 1.

(7) (a) How many zeroes does 100! end in?
 (b) What is the final non-zero digit in 100! ?

(8) Let $A = 4444^{4444}$. Let B be the sum of the (base 10) digits of A. Let C be the sum of the digits of B. What is the sum of the digits of C?

(9) Prove that the sequence (in base-10 notation)

$$11, 111, 1111, 11111, ...$$

contains no squares.