Math 194, problem set #2

For discussion October 8

1. Show that if the fraction a/b is expressed as a decimal number (where a, b are positive integers), it either terminates, or begins repeating after at most $b-1$ decimal places. (Hint: if you actually work out the long division, dividing a by b, what does it mean for the decimal expansion to repeat?)

2. The Fibonacci numbers are defined by the recurrence relationship

$$F_1 = 1 \qquad F_2 = 1 \qquad F_{n+2} = F_{n+1} + F_n \quad \text{for} \quad n = 1, 2, 3, \ldots$$

   Show

$$F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}$$

3. Inside a unit square, 101 points are placed. Show that some three of them form a triangle with area no more than .01.

4. Show that for $n \geq 6$ a square can be dissected into n smaller squares, not necessarily all of the same size.

5. The Euclidean plane is divided into regions by drawing a a finite number of straight lines. Show that it is possible to color each of these regions either red or blue in such a way that no two adjacent regions have the same color. (Putnam 1962)

6. Given any 5 distinct points on the surface of a sphere, show there exists a closed hemisphere that contains at least 4 of them. (Putnam 2002)

7. Let S denote an $n \times n$ lattice square, $n \geq 3$. Show that it is possible to draw a polygonal path consisting of $2n-2$ segments which will pass through all of the n^2 lattice points of S.

8. In how many ways can a $2 \times n$ square be tiled with 2×1 dominos?

9. Show that in any group of 6 people there are either 3 mutual acquaintances or 3 mutual strangers.

10. The numbers from 1 to 10 are arranged in some order around a circle. Show that there are three consecutive numbers whose sum is at least 17.