Math 194

November 19, 2015

- 1. (a) Prove that a triangle with sides of length (5,5,6) has the same area as a triangle with sides of length (5,5,8).
 - (b) Find an infinite set of other pairs of incongruent isoceles triangles with integer sides having equal area.
- 2. Show that you can cut any acute triangle into three pieces that can be rotated and translated (but not flipped) into a mirror image of the original triangle.
- 3. A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube? (Putnam 1998)
- 4. Prove that it is impossible for seven distinct straight lines to be situated in the euclidean plane so as to have at least six points where exactly three of these lines intersect and at least four points where exactly two of these lines intersect. (Putnam 1973)
- 5. Can one cover the whole plane, without overlaps, by squares with side length 1, 2, 4, 8, 16, ..., using each square at most 10 times? at most once?