Math 194

October 22, 2015

1. Show that
$$\sum_{k=0}^{n} \frac{1}{k+1}\binom{n}{k} = \frac{2^{n+1}-1}{n+1}.$$

2. Suppose $x_1, x_2, \ldots$ is a sequence of positive real numbers satisfying $x_{n+1} \leq x_n + 1/n^2$ for all $n \geq 1$. Prove that $\lim_{n\to\infty} x_n$ exists.

3. Show that
$$F_1 + F_2 + \cdots + F_n = F_{n+2} - 1$$
where F_n is the n-th Fibonacci number.

4. Evaluate
$$S_1 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

5. Express $\displaystyle\prod_{n=0}^{\infty}(1 + x^{2^n})$ as a rational function of x (a quotient of polynomials).