Math 194

Thursday, Oct. 15, 2015

1. Suppose $f(x)$ is a polynomial with real coefficients, and $a \in \mathbf{R}$. Show that $(x-a)^k$ divides $f(x)$ if and only if $f(a) = f'(a) = f''(a) = \cdots f^{(k-1)}(a) = 0$.

2. Consider all lines which meet the graph $y = 2x^4 + 7x^3 + 3x - 5$ in four distinct points, say (x_i, y_i), $i = 1, 2, 3, 4$. Show that

$$\frac{x_1 + x_2 + x_3 + x_4}{4}$$

   is independent of the line, and find its value. (Putnam 1977)

3. What is the remainder when the polynomial $x^{2009} - 2009$ is divided by $x^4 - 2$?

4. Show that there are no polynomials $f(x), g(x)$ such that $e^x = f(x)/g(x)$ for every x.

5. Suppose the polynomials $x^2 + ax + b$ and $x^2 + px + q$ have exactly one root in common. What is it?