Math 194

Thursday, Oct. 8, 2015

1. Show for positive integers a, b that if 7 divides $a^2 + b^2$ then it divides both a and b.

2. Find positive integers n and $a_1, a_2, \ldots, a_n$ such that

$$a_1 + a_2 + \cdots + a_n = 1979$$

   and the product $a_1 a_2 \cdots a_n$ is as large as possible. (Putnam, 1979)

3. (a) What are the last two digits of 3^{2009}?
   (b) What are the last two digits of 97^{2009}?

4. Prove that if $2n + 1$ and $3n + 1$ are both perfect squares, then n is divisible by 40.

5. Suppose that $P(x)$ is a polynomial with integer coefficients. If none of $P(1)$, $P(2)$, $P(3)$, …, $P(2009)$ is divisible by 2009, show that $P(x)$ has no integer roots.