

## Math 194

Thursday, Oct. 1, 2015

1. Show that  $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ .
2. Consider five points in the interior of a square of side length 1. Show that there is at least one pair of points a distance of at most  $1/\sqrt{2}$  apart. Show that  $1/\sqrt{2}$  is best possible.
3. Show that every number in the sequence

$$1007, 10017, 100117, 1001117, \dots$$

is divisible by 53.

4. Suppose 9 integer lattice points are given in  $\mathbf{R}^3$ . Show that there is at least one pair of these points such that the midpoint of the line segment joining them is also an integer lattice point.
5. Show that if you choose a set of ten numbers from the positive integers less than 100, then it contains two disjoint subsets whose sums are equal.