

## Math 194

Thursday, Sept. 24, 2015

1. Suppose 13 points in the plane are given, numbered  $P_1, P_2, \dots, P_{13}$ . Consider the 13 line segments  $P_1P_2, P_2P_3, \dots, P_{13}P_1$ . Is it possible that there is a straight line that crosses each segment (crossing means intersecting the segment in one point, not an endpoint)?
2. If  $A, B$ , and  $C$  are the 3 angles of a triangle, show that

$$\frac{\sin(A) + \sin(B)}{2} \leq \sin\left(\frac{A+B}{2}\right).$$

3. Show that

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} \leq 2\sqrt{n}.$$

4. Find all real numbers  $x$  that satisfy

$$\frac{3}{x-1} < \frac{2}{x+1}.$$

5. Suppose  $a$  is a real number and  $a + 1/a$  is rational. Prove that  $a^n + 1/a^n$  is rational for every integer  $n$ .