

## Math 194, problem set #8

For discussion December 5

- (1) A box in the shape of a rectangular solid with integer sides has a surface area that is equal to twice its volume – determine all possible dimensions for such a box. For example, if the box is  $2 \times 3 \times 6$  its volume is 36 and its surface area is 72.
- (2) A farmer, who sells grain, has a set of four weights that each weigh an integer number of pounds and a fair balance. She claims that she can weigh any integer number of pounds of grain up to a maximum of  $N$  using just these weights. What is the maximum value of  $N$  and what should the weights be?
- (3) The octagon  $P_1P_2P_3P_4P_5P_6P_7P_8$  is inscribed in a circle, with the vertices around the circumference in the given order. Given that the polygon  $P_1P_3P_5P_7$  is a square of area 5, and the polygon  $P_2P_4P_6P_8$  is a rectangle of area 4, find the maximum possible area of the octagon. (Putnam 2000)
- (4) Let  $s$  be any arc of the unit circle lying entirely in the first quadrant. Let  $A$  be the area of the region lying below  $s$  and above the  $x$ -axis and let  $B$  be the area of the region lying to the right of the  $y$ -axis and to the left of  $s$ . Prove that  $A + B$  depends only on the arc length, and not on the position, of  $s$ . (Putnam 1998)
- (5) Find the positive value of  $m$  such that the area in the first quadrant enclosed by the ellipse  $\frac{x^2}{9} + y^2 = 1$ , the  $x$ -axis, and the line  $y = 2x/3$  is equal to the area in the first quadrant enclosed by the ellipse  $\frac{x^2}{9} + y^2 = 1$ , the  $y$ -axis, and the line  $y = mx$ . (Putnam 1994)
- (6) A regular pentagon has side of length  $L$ . Compute the length of its diagonal. Your answer should be of the form  $a + b\sqrt{c}$  where  $a, b$  and  $c$  are rational numbers.
- (7) Circles with radii 12 and 4 are tangent as shown. A square is drawn inside the larger circle touching it and the smaller one as shown. What is the length of a side of the square?

