

Math 194, problem set #4

For discussion Thursday October 24

- (1) What is the remainder when the polynomial $f(x)$ is divided by $(x - a)^2$? by $x^2 - a$?
- (2) For which real values of p and q are the roots of the polynomial $x^3 - px^2 + 11x - q$ three successive integers? Give the roots in these cases.
- (3) Find, without a calculator, the smallest integer larger than $(\sqrt{3} + \sqrt{2})^6$.
- (4) (a) Determine all (a, b, c) such that the quadratic polynomial, $Q(n) = an^2 + bn + c$, assumes integer values for every integer n .
(b) Determine all (a, b, c, d) such that the cubic polynomial, $C(n) = an^3 + bn^2 + cn + d$, assumes integer values for every integer n .
- (5) If $P_n(x)$ denotes a polynomial of degree n such that $P_n(k) = \frac{1}{k}$ for $k = 1, 2, 3, \dots, n + 1$, determine $P_n(n + 2)$.
- (6) For which real numbers c is there a straight line that intersects the curve
$$y = x^4 + 9x^3 + cx^2 + 9x + 4$$
in 4 distinct points? (Putnam 1994)
- (7) Let k be a positive integer. Find all polynomials $P(x)$ with coefficients that are real numbers, satisfying the identity
$$P(P(x)) = P(x)^k.$$
- (8) For which ordered pairs of real numbers (b, c) do both of the roots of the quadratic polynomial $z^2 + bz + c$ lie inside the unit disk $\{z : |z| < 1\}$? (Putnam 1975)
- (9) Let $f(x)$ be a polynomial, and $a \neq b$. Suppose $f(x)$ leaves the remainder A when divided by $x - a$ and the remainder B when divided by $x - b$. Find the remainder when $f(x)$ is divided by $(x - a)(x - b)$.
- (10) Is there an infinite sequence of nonzero real numbers a_0, a_1, a_2, \dots such that the polynomial $a_0 + a_1x + a_2x^2 + \dots + a_nx^n$ has exactly n real roots?