Math 194

For discussion Tuesday, Nov. 13, 2012

1. For what positive x does the series

$$(x-1)+(\sqrt{x}-1)+(\sqrt[3]{x}-1)+\cdots+(\sqrt[n]{x}-1)+\cdots$$

converge? (The Wohascum County Problem Book, MAA, 1996)

2. Does the series $\sum_{n=1}^{\infty} \sin(\pi\sqrt{n^2+1})$ converge? (Gh. Sireţchi)

3. Show that

$$\sum_{n=1}^{9999} \frac{1}{(\sqrt{n}+\sqrt{n+1})(\sqrt[4]{n}+\sqrt[4]{n+1})} = 9.$$

(*Mathematical Reflections*)

4. Compute the product

$$\left(1-\frac{4}{1}\right)\left(1-\frac{4}{9}\right)\left(1-\frac{4}{25}\right)\left(1-\frac{4}{36}\right)\cdots$$

5. Compute the product

$$\prod_{n=1}^{\infty}(1+x^{2^n}).$$

6. Find a formula for the general term of the sequence

$$1,2,2,3,3,3,4,4,4,4,5,5,5,5,5,\ldots$$

(Graham, Knuth & Patashnik, *Concrete Mathematics*)

7. Define the sequence $a_0, a_1, a_2, \ldots$ by $a_0 = 0$, $a_1 = 1$, $a_2 = 2$, $a_3 = 6$, and

$$a_{n+4} = 2a_{n+3} + a_{n+2} - 2a_{n+1} - a_n, \quad \text{for } n \geq 0.$$

Show that n divides a_n for every $n \geq 1$.

(D. Andrica, *Timişoara Mathematics Gazette*)

8. Suppose $x_1, x_2, \ldots$ is a sequence of positive real numbers satisfying $x_{n+1} \leq x_n + 1/n^2$ for all $n \geq 1$. Prove that $\lim_{n\to\infty} x_n$ exists.

(De Souza & Silva, *Berkeley Problems in Mathematics*)

9. Prove that if a and b are relatively prime odd positive integers, then

$$\sum_{x=1}^{b-1} \left\lfloor \frac{ax}{b} \right\rfloor = \frac{(a-1)(b-1)}{2}.$$

10. Evaluate

$$\sum_{n=1}^{\infty} \frac{n}{3^n}.$$