Math 194, problem set #8

For discussion Tuesday November 29

(1) A box in the shape of a rectangular solid with integer sides has a surface area that is equal to twice its volume – determine all possible dimensions for such a box. For example, if the box is $2 \times 3 \times 6$ its volume is 36 and its surface area is 72.

(2) A farmer, who sells grain, has a set of four weights that each weigh an integer number of pounds and a fair balance. She claims that she can weigh any integer number of pounds of grain up to a maximum of N using just these weights. What is the maximum value of N and what should the weights be?

(3) The octagon $P_1P_2P_3P_4P_5P_6P_7P_8$ is inscribed in a circle, with the vertices around the circumference in the given order. Given that the polygon $P_1P_3P_5P_7$ is a square of area 5, and the polygon $P_2P_4P_6P_8$ is a rectangle of area 4, find the maximum possible area of the octagon. (Putnam 2000)

(4) Let s be any arc of the unit circle lying entirely in the first quadrant. Let A be the area of the region lying below s and above the x-axis and let B be the area of the region lying to the right of the y-axis and to the left of s. Prove that $A + B$ depends only on the arc length, and not on the position, of s. (Putnam 1998)

(5) Find the positive value of m such that the area in the first quadrant enclosed by the ellipse $\frac{x^2}{9} + y^2 = 1$, the x-axis, and the line $y = 2x/3$ is equal to the area in the first quadrant enclosed by the ellipse $\frac{x^2}{9} + y^2 = 1$, the y-axis, and the line $y = mx$. (Putnam 1994)

(6) A regular pentagon has side of length L. Compute the length of its diagonal. Your answer should be of the form $a + b\sqrt{c}$ where a, b and c are rational numbers.

(7) Circles with radii 12 and 4 are tangent as shown. A square is drawn inside the larger circle touching it and the smaller one as shown. What is the length of a side of the square?