Pre-Putnam List of Tools

General Methods

1. Establish convenient notation. Draw a picture.

2. Devise a plan for solving the whole problem, and then execute that plan.

3. Solve a simpler problem. Set a free parameter to a nice fixed value, start by looking at special/small cases, or consider the problem in fewer dimensions.

4. Prove by contradiction.

5. Use induction.

6. Work backwards, considering what your last steps could be.

7. Relate the problem, the conditions, or the unknown of the current problem to a problem that you've seen before.

8. Introduce an auxiliary element. Draw a new line segment or circle. Add zero or multiply by one in a helpful way. Find intermediate quantities when proving an inequality.

9. Solve a more general problem. Turn a numerical series into a power series or find something for all n when asked for a specific value. Consider a refinement.

Specific Techniques

1. Use the Pigeonhole Principle.

2. Look at parity or conditions modulo m.

3. Change variables to simplify expressions. Consider valuable substitutions.

4. Use calculus. Maximize functions. Use left and right-hand Riemann sums to approximate integrals (or vice versa).

5. Factor. Every positive integer can be expressed uniquely as a product of primes ($2006 = 2 \cdot 17 \cdot 59$). Every polynomial can be factored into linear factors over $\mathbb{C}$, or linear and quadratic factors over $\mathbb{R}$.

6. Use the fact that there can be no integer between consecutive integers.

7. Use inclusion-exclusion.

8. Look at the coefficients of polynomials or power series. You can write down the relationships between the coefficients and the roots of a polynomial.

9. Use the Arithmetic Mean-Geometric Mean inequality, $\frac{1}{2}(a+b) \geq \sqrt{ab}$ (For any number of positive real terms, their arithmetic mean is greater than or equal to their geometric mean).

10. Use Heron's Formula for the area of a triangle (Area $= \sqrt{s(s-a)(s-b)(s-c)}$, where a, b, and c are the side lengths and s is the semiperimeter).

11. If $f(x) = a - x$, then $f(f(x)) = x$. If $g(x) = 1 - \frac{1}{x}$ or $g(x) = \frac{1}{1-x}$, then $g(g(g(x))) = x$.

12. Use properties of binomial coefficients.

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\binom{n}{k} = \text{the number of subsets of size } k \text{ of an } n \text{ element set}$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

$$\sum_{k=0}^{n} \binom{n}{k} = (1+1)^n = 2^n = \text{the number of subsets of an } n \text{ element set}$$

For old Putnam exams and solutions, check out:

`http://www.unl.edu/amc/a-activities/a7-problems/putnamindex.shtml`

For Putnam practice exams with solutions from the University of Illinois, check out:

`http://www.math.uiuc.edu/~hildebr/putnam/mockputnam.html`