Math 194, problem set #4

For discussion Tuesday October 27

(1) What is the remainder when the polynomial $f(x)$ is divided by $(x-a)^2$? by x^2-a?

(2) For which real values of p and q are the roots of the polynomial $x^3-px^2+11x-q$ three successive integers? Give the roots in these cases.

(3) Find, without a calculator, the smallest integer larger than $(\sqrt{3}+\sqrt{2})^6$.

(4) (a) Determine all (a,b,c) such that the quadratic polynomial, $Q(n)=an^2+bn+c$, assumes integer values for every integer n.
(b) Determine all (a,b,c,d) such that the cubic polynomial, $C(n)=an^3+bn^2+cn+d$, assumes integer values for every integer n.

(5) If $P_n(x)$ denotes a polynomial of degree n such that $P_n(k)=\frac{1}{k}$ for $k=1,2,3,\ldots,n+1$, determine $P_n(n+2)$.

(6) For which real numbers c is there a straight line that intersects the curve

$$y=x^4+9x^3+cx^2+9x+4$$

in 4 distinct points? (Putnam 1994)

(7) Let k be a positive integer. Find all polynomials $P(x)$ with coefficients that are real numbers, satisfying the identity

$$P(P(x))=P(x)^k.$$

(8) For which ordered pairs of real numbers (b,c) do both of the roots of the quadratic polynomial z^2+bz+c lie inside the unit disk $\{z : |z|<1\}$? (Putnam 1975)

(9) Let $f(x)$ be a polynomial, and $a\neq b$. Suppose $f(x)$ leaves the remainder A when divided by $x-a$ and the remainder B when divided by $x-b$. Find the remainder when $f(x)$ is divided by $(x-a)(x-b)$.

(10) Is there an infinite sequence of nonzero real numbers $a_0,a_1,a_2,\ldots$ such that the polynomial $a_0+a_1x+a_2x^2+\cdots+a_nx^n$ has exactly n real roots?