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★**Algèbres non associatives et algèbres génétiques. (French)**
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The primary purpose of this work is to relate the concepts of genetics to those of nonassociative algebra and then to derive applicable results in the related algebraic systems. The material is divided into three main sections devoted to basic algebraic and arithmetic properties of algebras, motivational concepts from genetics, and a study of algebras derived from these genetic considerations.

In the first section a very careful treatment is given of various basic concepts and introductory material necessary to establish a structure theory. The extent of this treatment is indicated by a partial list of paragraph headings: power associative rings, finite-dimensional algebras, linearization and canonical bases, isotopic algebras, isotopy and associativity, isotopy and commutativity, duplication of algebras (a concept introduced by I. M. H. Etherington [Proc. Edinburgh Math. Soc. (2) **6** (1941), 222–230; MR0005113 (3,103b)]), idempotent and nilpotent elements, radicals, semisimple algebras, solvability and Jordan algebras.

The author discusses the Mendelian laws and some of the more recent developments in genetics in the second section. These involve the exceptions to the principle of independent assortment of pairs of genes, i.e., the supposition that certain pairs of genes are "linked" on a chromosome in such a way that the assortment in the progeny is more frequently the same as in the parents. Thus it can only be said that those genes lying in distinct linkage groups show independent assortment. However, even in the case of genes belonging to the same linkage groups the possibility of "crossovers" or reassortments is possible. The frequency of a given reassortment occurring in the progeny is a measure of the relative distance between genes in a linkage group. Thus if in the population the ratio of reassortments to non-reassortments is y to $1-y$ the value of y will depend on the proximity of the two locations of the genes on the chromosomes. If $y=\frac{1}{2}$ (for a pair of genes) then the two genes are considered to be independent.

These genetic considerations lead to the introduction of certain classes of algebras of which we shall mention only a few. Let A be a commutative algebra over F with a nontrivial homomorphism w (weight function) on A into V. Since A is finite-dimensional w can be extended to any scalar extension and hence can be defined for a generic element x of A. The algebra A is said to be a train algebra if the coefficient of x^q in the generic polynomial of right powers is $\beta_q w(x)^{r-q}$, where r is the degree of the polynomial and $\beta_q\in F$.

This class of algebras contains those classes derived directly from genetic considerations. Some results for low dimension on the radical and arithmetic properties are obtained. Particular attention is given to the subalgebras generated by the elements of weight 1, since these elements represent populations having a certain distribution of genetic factors. The importance of the existence of idempotents is well attested to in the structure theory of these algebras (see, for example, R. D. Schafer [Amer. J. Math. **71** (1949), 121–135; MR0027751 (10,350a)]); however, this importance is increased by the recognition that the existence of an idempotent represents a state of equilibrium in the population.

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