Complex Analysis Math 220A

Midterm Sample Exam

Problem 1.

Find the radius of convergence for the series:

$$\sum_{n=1}^{+\infty} \frac{z^{2n}}{n!} \quad \text{and} \quad \sum_{n=1}^{+\infty} \frac{z^{n!}}{2n}$$

Problem 2.

Prove that

$$\cos\frac{\pi}{11} + \cos\frac{3\pi}{11} + \cos\frac{5\pi}{11} + \cos\frac{7\pi}{11} + \cos\frac{9\pi}{11} = \frac{1}{2}.$$

(Hint: use complex numbers!)

Problem 3.

At which points the following functions are $\mathbb{C}$-differentiable?

$a)$ $f(z) = x^2 + y^2 + 2ixy$, where $z = x + iy$,

$b)$ $f(z) = z\ Rez.$

Compute their derivatives at these points where they are $\mathbb{C}$-differentiable.

Problem 4.

Suppose that $\gamma : [0,1] \to U$, where $U = \mathbb{C}\ \{0,1\}$, is a closed ($\gamma(0) = \gamma(1)$) smooth curve, and for any $f \in \mathcal{O}(U)$ we have

$$\int_\gamma f(z)dz = 0.$$

Does it imply that γ is homotopic to a point?

Problem 5.

Let $p(z)$ be a polynomial. Suppose that $p(z) \neq 0$ for $\mathrm{Re}(z) > 0$. Prove that $p'(z) \neq 0$ for $\mathrm{Re}(z) > 0$.