MONODROMY MATRICES OF 1D DIFFERENTIAL OPERATOR OF ORDER 4

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Let $p(x)$ and $q(x),\ x \in [0, \pi]$, be a pair of real-valued functions satisfying conditions

$$\int_0^\pi (|p'(x)|^2 + |q(x)|^2)dx < \infty, \tag{1}$$

and let $\mathcal{L}$ be the differential operator

$$\mathcal{L} = \frac{d^4}{dx^4} + \frac{d}{dx}\ p(x)\frac{d}{dx} + q(x), \quad x \in (0, \pi). \tag{2}$$

We denote by $U(x, \lambda), \lambda \in \mathbb{C}$, the *fundamental* matrix of the equation $\mathcal{L}u = \lambda u$, i.e., we set

$$U(x, \lambda) = ||u_k^{(j-1)}(x, \lambda)||_{k,j=1}^4,\ U(0, \lambda) = I,$$

with $u_k(x, \lambda), k = 1, ..., 4$, being solutions to the above equation. It is well known that the *monodromy* matrix $U(\pi, \lambda)$ contains, one way or another, all data related to the boundary problems generated by L in the interval $[0, \pi]$.

We describe the set of all 4×4 matrices $U(\lambda), \lambda \in \mathbb{C}$, which are the monodromy matrices of operators (2) restricted by condition (1).

The main tool to obtain such a description is the *transformation* operator introduced by Z.Leibenzon [1]-[2].

References

[1] Z.Leibenzon, Trudy Mosc. Math. Ob.,Trans.MMS, 1966, v.15, 78-163.

[2] Z.Leibenzon, Trudy Mosc. Math. Ob.,Trans.MMS, 1971, v.25, 13-61.