

Sample Midterm 1

Problem 1. There are 12 members on the board of directors for the Newport General Hospital

(a) If they must elect a chairperson, first vice chairperson, second vice chairperson and a secretary, how many different slates of candidates are possible?

(b) If they must form an ethics subcommittee of four members, how many different subcommittees are possible?

Problem 2. When you give a casino \$5 for a bet on the "pass line" in the game of craps, there is a $244/495$ probability that you will win \$5 and a $251/495$ probability that you will lose \$5. What is your expected value? In the long run, how much do you lose for each dollar bet?

Problem 3. Mars, Inc. claims that 10% of its M&M plain candies are blue, and a sample of 1000 candies is randomly selected.

(a) Find the mean, and standard deviation for the number of blue candies in such groups of 1000.

(b) What is the probability of getting more than 150 blue ones?

Problem 4. Replacement times for CD players are normally distributed with a mean of 7.1 years and a standard deviation of 1.4 years.

(a) Find the probability that a random selected CD player will have a replacement time less than 8 years.

(b) Assume that you take a sample of 9 CD players. Find the probability that the sample will have a mean replacement time less than 8 years.

(c) If you want to provide a warranty so that only 2% of the CD players will be replaced before the warranty expires, what is the time length of the warranty?

Problem 5. In crash tests of 15 Honda Odyssey minivans, collision repair costs were found to have a distribution that is roughly bell shaped, with a mean of \$1786 and a standard deviation of \$937. Construct a 99% confidence interval for the mean repair cost in all such vehicle collisions.

Cheat Sheet

◇ Mean:

$$\mu = \int_{-\infty}^{\infty} x f(x) dx \quad (\text{Continuous}), \quad \mu = \sum_x x p(x) \quad (\text{Discrete})$$

◇ Variance:

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2 \quad (\text{Continuous}), \quad \sigma^2 = \sum_x x^2 p(x) - \mu^2 \quad (\text{Discrete})$$

For frequency distribution tables:

◇ Binomial:

$$X \sim B(n, p) \Leftrightarrow Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k} = \frac{n!}{k! \times (n-k)!} p^k (1-p)^{n-k}$$

◇ Poisson:

$$X \sim Poisson(\lambda) \Leftrightarrow Pr(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$$

◇ Confidence Interval:

✓ For known variance (or large sample size) we get the $100(1-\alpha)\%$ confidence interval

$$CI = \left(\bar{X} - z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}}, \bar{X} + z_{\alpha/2} \times \frac{\sigma}{\sqrt{n}} \right)$$

✓ For unknown variance (or small sample size) we get the $100(1-\alpha)\%$ confidence interval

$$CI = \left(\bar{X} - t_{\alpha/2} \times \frac{s}{\sqrt{n}}, \bar{X} + t_{\alpha/2} \times \frac{s}{\sqrt{n}} \right)$$