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Asymptotics, Stability, and Reliability of Random Systems

Brief Summary of Research Interests

In broad terms, my research interests are at the intersection of stochastic processes, dynamical systems, and ergodic theory. More specifically, I tend to be interested in various asymptotic issues of random differential and integro-differential systems. Below is a more detailed description of several directions of my research interests, and the outline of the program for future research in some of them.

Identification of Stratified Processes

Several new processes whose evolution spaces vary their dimensions (stratified processes), were studied in probability literature during the last decade. They include fiber Brownian motion introduced by Bass and Burdzy [1] as a process which switches between two-dimensional and one-dimensional evolution, a Markov process on a 'whiskered sphere' described by Sowers in [28], processes on trees like the Walsh process, the Evans process, spider martingales, and Brownian snakes [7, 31], some processes described by Freidlin and Wentzell [8]. Recently, Evans and Sowers [6] made certain generalizations about possible structures of the stratified spaces as evolution spaces for the graph-valued Markov processes, they also described the generators of such processes.

I have identified one of the Markov processes on a space with stratified geometry as a weak limit of a Hamiltonian system with a degenerate Hamiltonian and multiple time scales [25, 26]. I am interested in exploring how to explain heuristically the dynamics behind some of these processes. The heuristic picture is more visible when a process is described via a system of differential equations. For the processes I am considering, the fine analysis near the 'troublesome' points, i.e., those points where the process moves from one strata to another, has to be performed in a set of finer coordinates, so-called Hasminskii's coordinates. The system of resulting equations includes the Dirichlet-to-Neumann boundary conditions. It is not always clear how to match these conditions with the description of the cores of the generators of the corresponding stratified processes. This is one of the problems I am currently working on.

The issue of identification of stratified processes as limits of certain Markovian systems currently interests me mostly from the prospective of studying the rate of their weak convergence to the limit.

Rate of Convergence to Stratified Processes

I am especially interested in exploring the application of the Wasserstein-type minimal metrics (described, for example, in [27]) to the issues of stability and reliability of stochastic systems. These metrics are valuable for my purpose because they metrize weak convergence, and also because they are not very demanding with respect to the underlying geometric structures of the image-spaces, so many relevant results work for general Polish spaces. Wasserstein metrics have been previously applied to study the rates of convergence of diffusion processes, mainly by constructing embedded discrete-time processes, as for example done by Kanagawa in [11], and almost exclusively for one-dimensional problems which can be reformulated in terms of pseudo-inverse functions [30]. Such reformulations in pseudo-inverse functions then allow for an application of the Fokker-Planck equation, like in recent work of Carillo and Toscani [3]. The connections between the Fokker-Planck and the Wasserstein metric have also been previously explored. For example, it is known [9] that the Fokker-Planck equation can be interpreted as the steepest descent for a free energy related to Boltzmann-Gibbs entropy with respect to the Wasserstein metric.

The complex structure of the stratified space of the systems of my current interest do not allow for an explicit reformulation in terms of pseudo-inverse functions, and the familiar Fokker-Planck method cannot be applied. Applying Strassen's duality argument, I recently showed [24] that the rate of convergence to some stratified processes can be controlled by the rates of convergence of related processes on simpler (non-stratified) structures. After analyzing the related classical case on a cylinder, where an explicit computation is possible, it becomes evident that the estimates obtained in [24] for the related stratified case are not sharp yet, but the corresponding calculation in the classical case produces a discrepancy of the smallest possible order ε^{α} with any $\alpha > 0$. I would also like to answer the question whether it is possible to give the estimates for the rate of weak convergence in Wasserstein-type metric by making use of Hasminskii's coordinates.

Reliability of Random Differential and Integro-Differential Systems

For some time I have been interested in those asymptotic issues of random differential and integro-differential equations which may be useful from the immediate engineering standpoint. In contrast to the aspects of my research dealing with the convergence to stochastic stratified processes, the limiting processes here are deterministic. The literature on this topic is vast, and the interest is usually driven by the abundance of applications.

One work that I have done in the past was related to the study of composite materials whose molecular-level-structure has a periodically alternating pattern. Such materials are known to behave in a different way than non-composite materials with the same effective characteristics. Processes in such composite materials are usually described by systems with fast-oscillating coefficients. For one of the classes of such random systems, I have obtained the explicit large-deviation estimates [21]. Also, for a different class of random systems, satisfying a type of mixing condition, explicit large-deviation estimates were given in [20].

I currently consider (work in progress) a class of integro-differential equation which arise naturally in those cases where the current behavior of a system depends not only on its present state, but also on the whole history of its development since some fixed time. The systems of my interest also include certain differential systems with delay. The integro-differential operators of my concern involve integration in time, and are different from another popular type of integro-differential operators, where integration is done in a space variable, like those for example in Hamilton-Jacobi-Bellman equations. For such random integro-differential equations, I consider their corresponding averaged deterministic equations and obtain explicit large-deviation-type estimates. The processes treated by this method include a wide variety of Markov processes, in particular they include ergodic processes satisfying Doeblin's condition, and many of the stationary Gaussian processes. This work is generalizing [20], and also treats the multidimensional case.

Asymptotics of Multiparameter Processes

In the past, I studied the systems which involve multiparameter processes mostly from the engineering, or statistical point of view. In collaboration with Bondarev [2], we obtained an averaging result for a random field described by a stochastic Goursat problem.

The problem of my current amusement is of a somewhat different nature, and it is just one of many open problems about the multiparameter Gaussian processes. I would like to find a workable approach to showing the large-deviation result for a Wiener sausage of a Brownian sheet.

A Wiener sausage is the path traced by a ball whose center moves along a Brownian trajectory. It was first considered in the mid-60's by Kesten, Spitzer, and Whitman. Most of the asymptotic results about Brownian sausage were motivated by physical problems. For example, in 1974, Kac and Luttinger [10] used some of the conjectured results about the volume of the Brownian sausage to discuss Bose-Einstein condensation in the presence of impurities. The proof of those results was soon given by Donsker and Varadhan [4, 5]. These results then were extended by many authors and expanded in many directions. An elegant way to prove the large-deviation result for a Wiener sausage for Brownian motion was found by Sznitman [29], who handled it as an application of his method of enlargement of obstacles.

An apparently easier related problem—one which is, however, still open—is the large deviation for the additive Brownian motion. How closely these two problems are connected is visible for example from the paper by Khoshnevisan and Shi [12].

Central Limit Theorem for Ergodic Markov Processes

Although I am currently not prioritizing this direction, I have been intensely interested in some aspects of the Central Limit Theorem in the past. The series of short papers [14–19] in collaboration with Shurenkov, and paper [22] treat the asymptotic normality of certain functionals of Markov chains. The results obtained are equivalent to some of the results described by Nummelin in [23], but proved using a different, so-called analytic approach, avoiding the assumption that the second moment of the regeneration times is finite. Similar results can be found in [13].

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