THEOREMS FOR THE FINAL

1. (Wed, Week 6) If G is a cyclic group, say $G = \langle a \rangle$ of order s and $d = \gcd(r, s)$ then

(a) $\langle a^r \rangle = \langle a^d \rangle$;

(b) $\langle a^r \rangle$ has exactly $\frac{s}{d}$ elements.

(Remark: $\gcd(r, s)$ is the greatest common divisor of r and s.)

2. (Mo, Week 7) If G is a cyclic group, say $G = \langle a \rangle$, then

(a) if G is infinite then G is isomorphic to $\langle \mathbb{Z}, + \rangle$;

(b) if G is finite of order s then G is isomorphic to $\langle \mathbb{Z}_s, +_s \rangle$.

3. (Wed+Fri, Week 7) Let $f : (G_1, *_1) \to (G_2, *_2)$ be a homomorphism.

(a) If H is a subgroup of G_1 then $H' = f[H]$ is a subgroup of G_2;

(b) $f(e_{G_1}) = e_{G_2}$ and $f(a^{-1}) = (f(a))^{-1}$;

(c) If K is a subgroup of G_2 then $K' = f^{-1}[K]$ is a subgroup of G_1.

(d) f is injective if and only if $\mathrm{Ker}(f) = \{e_{G_1}\}$.

(Remark: $f[H] = \{f(x) | x \in H\}$ and $f^{-1}[K] = \{x \in G_1 | f(x) \in K\}$.)

4. (Wed, Week 8) Formulate Cayley's Theorem and give its proof.

5. (Wed, Week 9/Mo, Week 10) Every permutation $\sigma \in S_n$ can be expressed as a product of disjoint cycles. Every permutation $\sigma \in S_n$ can be expressed as a product of transpositions.

6. (Wed, Week 10) Let $\sigma \in S_n$. If σ can be expressed as a product of even number of transpositions, then it cannot be expressed as a product of odd number of transpositions.