

SAMPLE FINAL

Structure of the test.

There will be five items. Each of the first four items will be worth 10 points, the fifth item will be worth 20 points. Thus, the total number of points will be 60. The first two items will be on theorems and **proofs** we did in the course. The theorems tested can be downloaded from the link "Theorems for the final". The three remaining items will contain computational problems of the same kind as in the following sample (the input data, of course will be different):

Item 3. Two computations regarding subgroups of cyclic groups. (5+5pt)

Item 4. Two computations regarding homomorphisms. (5+5pt)

Item 5. Three computations regarding permutations (5+8+7pt)

Sample Problems.

Item 3. (a) (5pt) Find all subgroups of \mathbb{Z}_{12} and draw their subgroup diagram.

(b) (5pt) Find the subgroup of \mathbb{Z}_{12} generated by the set $\{4, 6\}$.

Item 4. (a) (5pt) Given is a homomorphism $f : \mathbb{Z} \rightarrow \mathbb{Z}_6$ such that $f(1) = 2$. Find $f(25)$, $\text{Ker}(f)$ and $\text{rng}(f)$. (Remark: $\text{rng}(f) = f[\mathbb{Z}]$.)

(b) (5pt) Given is a homomorphism $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$ such that $f(\langle 1, 0 \rangle) = 1$ and $f(\langle 1, 1 \rangle) = 2$. Find $f(\langle 13, 5 \rangle)$ and $\text{Ker}(f)$.

Item 5. (a) (5pt) Given is the following permutation $\pi \in S_8$:

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 4 & 1 & 8 & 3 & 2 & 6 & 7 \end{pmatrix}.$$

Express π both as a product of disjoint cycles and a product of permutations. Determine $\text{sgn}(\pi)$.

(b) (8pt) Given are the following permutations $\sigma, \sigma' \in S_5$.

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 2 & 1 & 3 \end{pmatrix} \quad \sigma' = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 2 & 1 & 4 \end{pmatrix}.$$

Compute $\langle \sigma \rangle$, σ^{50} and $\sigma\sigma'$.

(c) (7pt) Given is a homomorphism $f : \mathbb{Z}_4 \rightarrow S_4$ such that $f(1) = (1, 3)(2, 4)$. Find $f(3)$ and $\text{Ker}(f)$.