

## Homework 8

### P371 Problem 1

For present value, we define  $i$  by  $E[(1 + i_t)^{-1}] = (1 + i)^{-1}$

$$(10.9) \quad E[a^{-1}(n)] = (1 + i)^{-n}$$

$$\begin{aligned} E[a^{-1}(n)] &= E \left[ \frac{1}{\prod_{t=1}^n (1 + i_t)} \right] = E \left[ \prod_{t=1}^n \frac{1}{(1 + i_t)} \right] \\ &= \prod_{t=1}^n E \left[ \frac{1}{(1 + i_t)} \right] \quad \text{for independence} \\ &= (1 + i)^{-n} \end{aligned}$$

### P371 Problem 2

$$(10.11) \quad E[a_n] = a_n i$$

$$\begin{aligned} E[a_n] &= E \left[ \sum_{t=1}^n \prod_{s=1}^t \frac{1}{(1 + i_s)} \right] = \sum_{t=1}^n E \left[ \prod_{s=1}^t \frac{1}{(1 + i_s)} \right] \\ &= \sum_{t=1}^n \prod_{s=1}^t E \left[ \frac{1}{(1 + i_s)} \right] \quad \text{for independence} \\ &= \sum_{t=1}^n \prod_{s=1}^t (1 + i)^{-1} = \sum_{t=1}^n (1 + i)^{-t} \\ &= a_n i \end{aligned}$$