Take Home Math 271C, 2008 Turn in Monday June 16th.

Problem #1

Assume the Black and Scholes model and that the payoff of an option corresponding to a *straddle*:

$$h(s) = \begin{cases} -s & \text{for } \ s < K; \\ s - 2K & \text{for } \ s \geq K \end{cases}$$

with $0 < K$. Give the explicit form for the option price.

Problem #2

Set up the American call with dividends as a linear complementarity problem. Can you solve it? What if the dividend is zero?

Problem #3

Consider the stochastic process

$$X_t = \gamma t + cB_t + J_t$$

with γ, c fixed parameters B_t standard Brownian motion and J_t a compound Poisson process with intensity λ and jump distribution F. Describe how you would create realizations of this process with a computer. Extra credit: write a program that simulates the process and plot some realizations.

Problem #4

Let h be a bounded function. Define the problem that gives the expression for $\mathbb{E}[h(X_t)]$.

Problem #5

(A)

Consider the stochastic process

$$dS_t = \alpha(t)S(t)dt + \sigma(t)S(t)dB(t)\,,$$

Find an explicit expression for $S(t)$.

(B) Suppose the model for a traded price process is the OU model with parameters (α, m, σ) for the rate of mean reversion, long term mean and standard deviation respectively. Assume a constant interest rate, r, and a European contract payoff function being the identity (the price) and find the price for this contract.