

Math 230B, problem set #3

- (1) Determine $\text{Gal}(\mathbf{Q}(\sqrt{6}, \sqrt{11})/\mathbf{Q})$. What are the automorphisms, and which familiar group is it?
- (2) Determine the Galois group of $x^3 - 7x + 3 \in \mathbf{Q}[x]$. Which familiar group is it?
- (3) If p is prime, show that the Galois group of $x^p - 2$ is isomorphic to the group of matrices
$$\left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} : a, b \in \mathbf{F}_p, a \neq 0 \right\}.$$
- (4) Suppose $f(x) = x^3 + ax + b \in F[x]$ for some finite field F , and let E be the splitting field of f . What are the possibilities for $\text{Gal}(E/F)$? What does this tell you about the discriminant of f ?
- (5) What is the irreducible polynomial for $\sqrt{3} + \sqrt{5}$ over \mathbf{Q} ?
- (6) Suppose that $f \in \mathbf{Q}[x]$ is an irreducible polynomial of degree 4, with Galois group S_4 . Let θ be a root of f . Show that $\mathbf{Q}(\theta)/\mathbf{Q}$ is an extension of degree 4 with no intermediate fields.