

Once Upon a Number: The Hidden Mathematical Logic of Stories

Reviewed by Colin Adams

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John Allen Paulos
Basic Books, 1998
ISBN 0-465-05158-8
224 pages, \$23.00

A few years ago I was trying to get a motion passed on the floor of a faculty meeting. I stood up and listed the six reasons, one after the other, that demonstrated that the faculty should vote for the motion. Then I sat down, confident in the irrefutable logic of my arguments. I assumed that the faculty, who are individually intelligent, would certainly see my points when stated in these starkly obvious terms. A nonscience colleague then stood up and, in a long meandering speech, gave one reason why the motion should not be passed. Someone else explained why that point was invalid. My opponent rose again and took another fifteen minutes to make a second point. This also was refuted. But for each refutation he had a fifteen-minute response. By the end, my colleague had held the floor for at least 80 percent of the discussion. When the vote was finally taken, the motion was defeated.

A mathematician's traditional rigorous, concise style of presenting material can be a disaster when utilized on a general audience. It is the drink of water out of a fire hose. The vast majority of people prefer to have a mathematical idea (or any other

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idea for that matter) explained in a variety of ways and from a variety of viewpoints, with lots of filler to give them time to absorb the idea slowly.

In his best-selling book *Innumeracy*, John Allen Paulos explained his attempts to grapple with his own tendency toward terseness: "I have a difficult time writing at length about anything. Either my mathematical training or

my natural temperament causes me to distill the crucial points and not to dwell (I want to write 'dither') over side issues or contexts or biographical detail. The result, I think, is clean exposition, which can nevertheless be intimidating to people who expect a more leisurely approach."

Paulos is best known for his two books, *Innumeracy* and *A Mathematician Reads the Newspaper*. Both of these books are fun reads. And both of these books have great titles. In one and five words respectively, they conjure up the entire contents of the books.

Paulos's new book does not have such a title. *Once Upon A Number: The Hidden Mathematical Logic of Stories* does not explain the book. But as it turns out, this is not an easy book to explain. In some sense it is an expansion of several themes

that have occurred in Paulos's previous books. Specifically, he is interested in the difference, as he says, between narrative and numbers, between stories and statistics. How do we synthesize the individual anecdotes and the informal discourse with the statistical overview, with the hard cold numbers? That still does not give one much of a sense of where this book is likely to go, since this is not terrain that has been much explored previously.

The book consists of an introduction and five interconnected essays. Each essay investigates a different aspect of the relationships between stories and statistics. Essay does seem the best word to utilize here, as opposed to chapter or treatise, as there is no well-defined thesis that is being argued. Each essay defies easy categorization and works its way over the landscape without a specific goal. Although the landscape is engrossing, readers may become frustrated by the apparent lack of forward progress. What follows is a detailed outline of the first essay to give a sense of how the book is written and shorter descriptions of the other essays.

In the first essay, "Between Stories and Statistics", Paulos notes that statistics grew out of informal stories and anecdotes. Concepts such as the mean, mode, variance, correlation, and others had precursors in such everyday words as usual, standard, stereotypical, peculiar, strange, association, and relation. In fact, many of our behaviors are dictated by an intuitive sense of statistics. As he says, "Admit it or not, we are all statisticians, as when we make grand inferences about a person from that tiny sample of behavior known as a first impression."

Unfortunately, we often lose sight of the story that generated the statistics. Paulos derides the use of statistics given out of context and emphasizes the care with which such statistics must be interpreted, a theme that appeared in both of the other books mentioned. He gives several examples where the statistics taken out of context are difficult to interpret, including the "birth effect", the psychological theory which states that the order of birth among siblings plays an important role in subsequent personality development.

Paulos gives the stimulating example (a slight variation of which appeared in *Innumeracy*) of an individual who sends out letters predicting the outcome of a sporting event, half predicting one result and the other half predicting the opposite. Then when the result is known, he sends another set of letters to those to whom it appeared his first prediction was correct. After repeating this process a few more times, he has a small set of people who believe him a genius or soothsayer. This story, to which Paulos adds various twists and turns, brings him to tree diagrams and their potential use in literature. From here we take a quick ride through

correlation and the fact that if the number of traits considered is larger than the number of individuals considered, statistical results will be meaningless. As he does throughout the book with the various mathematical concepts, Paulos asks how this principle might affect us in our daily interactions. In the real world the number of people we know is relatively small and the number of traits we consider is relatively large. Hence we may often see correlations where none exist.

In fact, my favorite aspect of the book is the author's willingness to take a mathematical principle and consider how it might apply in complex systems of interaction amongst people.¹ Although highly speculative, these considerations are always thought-provoking. As he himself says later on, "I am aware that part of what is written here may be dismissed as an unholy mixture of discordant fields; even I think this on Tuesdays and Saturdays. Nevertheless, on the other five days I think it is well worth a scientist's effort to try to explore the borderland between these disparate cultures."

The first essay also includes a discussion of stereotypes and the circumstances under which they are appropriate. This allows Paulos the opportunity to discuss the Op Ed piece he wrote for the *New York Times*, in which he stated that he had suspected the Unabomber was probably a mathematician, and the brouhaha that resulted from that statement.

The second essay is entitled "Between Subjective Viewpoint and Impersonal Probability". The focus here is on the psychological aspects of our interpretations of the world. The explicit examples are particularly interesting, including a discussion of the Bible Code and the O. J. Simpson trial. This is followed by "Between Informal Discourse and Logic", wherein Paulos considers the informal (intensional) logic of stories and contrasts it with the formal (extensional) logic of mathematics. An appendix on "Humor and Computation" is included, which is based on a talk Paulos gave in 1995 and on his two books *Mathematics and Humor* and *I Think, Therefore I Laugh*. Although not clearly relevant, the appendix does have some interesting points to make.

The fourth essay is entitled "Between Meaning and Information". Here is an opportunity to dip into information theory, cryptography, and complexity. Paulos touches on Ramsey's theorem and Smale's horseshoe map and the roles they may play in our day-to-day lives. The final essay is entitled "Bridging the Gap". Paulos discusses how we can over-

¹Well, actually, my very favorite part of the book is the image he describes of himself and his brother traversing the neighborhood in their underwear while throwing darts at trees, with Paulos triumphant in the fact he has swim trunks under his underwear.

come the divide between stories and statistics, but with caveats as to the dangers inherent in such an endeavor.

Paulos ends by saying, "How we can maintain a place for the individual, protected from the overweening claims of religion, society, and even science, is an increasingly important unsolved problem. Its solution, I have no doubt, will require simply and pragmatically accepting the indispensability of both stories and statistics and of their nexus, the individual who uses and is shaped by both. The gap between stories and statistics must be filled somehow by us."

Paulos sprinkles the book with quotes and connections and with quips from his favorite comedians. In some cases he is reaching a bit, as for instance when he says that "the word context is obtained by conjoining conte, which means a short tale or adventure, with xt, the most commonly used variables in statistics and mathematics as a bridge between the two worlds." Paulos acknowledges it is a stretch, but he includes it anyway.

Although he can verge on the didactic, most of the time he is eloquent: "In any case, the inexhaustible source of information is the unmediated world out there. By intelligently reducing parts of it to a formal calculi and systems, we tame larger and larger tracts. Still, even when we carry out the reduction thoughtfully, we bring order only to our trim hedgerow in the celestial landscape. Our cognitive homes generally are as unnaturally neat and comfortable as our physical ones."

Unfortunately for anyone who has read *Innumeracy* and *A Mathematician Reads the Newspaper*, many of the ideas and the specific examples in this book are repeats from those two books, sometimes with minor variation. These include the soothsayer example, the complexity of a sequence of 0's and 1's, regression to the mean, Type I and II errors in statistics, false identifications of witnesses, and others.

Another flaw is that the serpentine nature of the presentation sometimes causes confusion. For instance, Paulos utilizes Bayes's theorem and conditional probability on page 50 but does not explain them until page 69. He also has a tendency to make up probabilities rather than to search out the actual numbers, as when he guesses the probability that an adult weighs less than 130 pounds or the percent of those people who speak Spanish and who are from Spain. It would be more interesting to know the actual quantities.

The book *Innumeracy* gave one the feeling of being let loose in a candy store, where the shelves are stacked with bins and each bin contains a new surprising taste treat. One was almost overwhelmed with a surfeit of interesting examples. *A Mathematician Reads the Newspaper* was more like a stroll down a city block, where one could peer in the windows or enter the variety of unusual stores

along the way. *Once Upon a Number* is more like a canoe ride down a meandering river. The pace is leisurely and the goal is not to reach any particular destination. There are eddies and backwashes, but it is a pleasant ride nonetheless. Paulos has succeeded in slowing down his exposition and decreasing the information-per-number-of-characters ratio. Whether or not that is a benefit will depend on the individual reader.

There are no definitive questions when examining such amorphous topics, and therefore there can be no definitive answers. For mathematicians who expect such, this book will be a disappointment. But one has to respect Paulos for going where mathematicians rarely have the courage to go. He is willing to take statistical and mathematical principles and suggest connections with the complicated systems that make up our everyday world, and though these connections are perhaps tenuous, they are stimulating nonetheless.