meyberg 47-48 marked. ref

Use the same kind of argument to show that the standard imbedding of $K^{(p,q)}$ is the Lie algebra of $(p+q)\times(p+q)$ skew symmetric matrices over F.

6.3. Let $\mathcal{T}_i$ $(i = 1,2)$ be Lie triple systems and $\mathcal{L}_i = \mathfrak{H}_i \oplus \mathcal{T}_i$ the corresponding standard imbedding. If $\phi : \mathfrak{H}_1 \to \mathfrak{H}_2$ and $\eta : \mathcal{T}_1 \to \mathcal{T}_2$ are Φ-linear maps, then $\lambda : \mathcal{L}_1 \to \mathcal{L}_2$ defined by $\lambda(H \oplus x) = \phi(H) \oplus \eta(x)$ is obviously Φ linear.

<u>Lemma 3.</u> $\lambda : \mathcal{L}_1 \to \mathcal{L}_2$ <u>is a Lie algebra homomorphism, if</u> $\phi : \mathfrak{H}_1 \to \mathfrak{H}_2$ <u>is a Lie algebra homomorphism and</u>

(i) $\phi\, L_1(x,y) = L_2(\eta x, \eta y)$

(ii) $\eta H = \phi(H)\eta$

(L_i is the left multiplication of $\mathcal{T}_i$.)

<u>Proof.</u> Easy exercise. Exercise 4

A linear map $\eta : \mathcal{T}_1 \to \mathcal{T}_2$ is an L.t.-homomorphisms, if

$$\eta[xyz] = [(\eta x)(\eta y)(\eta z)] \quad \text{, or equivalently}$$

$$\eta L_1(x,y) = L_2(\eta x, \eta y)\eta$$

If $\eta : \mathcal{T}_1 \to \mathcal{T}_2$ is an L.t.-isomorphisms, then according to lemma 3, the map

Exercise 5: Prove the assertions in this box, i.e. 1, 2, 3, 4

$$\Lambda : H \oplus x \mapsto \eta H \eta^{-1} \oplus \eta x$$

is an 1 isomorphism of $\mathcal{L}_1$ onto $\mathcal{L}_2$. Obviously 2 Λ commutes with the main involutions, i.e. $\Lambda\Theta_1 = \Theta_2\Lambda$. If conversely $\Lambda : \mathcal{L}_1 \to \mathcal{L}_2$ is an isomorphismus such that $\Lambda\Theta_1 = \Theta_2\Lambda$ and if $1/2 \in \Phi$ then we get that the 3 restriction of Λ to $\mathcal{T}_1$ maps onto $\mathcal{T}_2$, hence is an 4 L.t.-isomorphism.

The following trivial observation is quite useful for applications. Assume η is an automorphism of $\mathcal{T}$, $\eta^2 = \text{id}$. Then $\Lambda : H + x \mapsto \eta H \eta + \eta x$

is an automorphism of $\mathcal{L}$ and $\Lambda^2 = \mathrm{id}$. Hence the (-1)-eigenspace of $\mathcal{L}$, i.e. $\mathcal{L}_- = \{X \in \mathcal{L}, \Lambda X = -X\}$ is a L.t.s. (6.1 ex. 2), which is (obviously) in most cases quite different from $\mathfrak{T}$, but which has, in certain cases, the same (isomorphic) standard imbedding as $\mathfrak{T}$, namely $\mathcal{L}$.

6.4. From now on we assume $1/2 \in \Phi$. In this case $x \in \mathfrak{T}$, iff $\Theta x = x$ (see theorem 1). We shall derive a rather strong connection between ideals in a L.t.s. $\mathfrak{T}$ and ideals in its standard imbedding $\mathcal{L}$.

Since $\mathcal{L}$ is a Lie algebra with involution Θ we are mainly interested in Θ-invariant ideals.

If $\mathfrak{K}$ is any Θ-invariant ~~submodule~~ subspace of $\mathcal{L}$, then $H \oplus x \in \mathfrak{K}$ implies $-H \oplus x \in \mathfrak{K}$, consequently $\mathfrak{K} = \mathfrak{H} \cap \mathfrak{K} \oplus \mathfrak{T} \cap \mathfrak{K}$. Conversely, any ~~submodule~~ subspace of this type is Θ-invariant. Let $\mathfrak{K} = \mathfrak{M} \oplus \mathfrak{U}$ be a Θ-invariant ~~submodule~~ subspace of $\mathcal{L}$ ($\mathfrak{M} \subset \mathfrak{H}$, $\mathfrak{U} \subset \mathfrak{T}$). $\mathfrak{K}$ is an ideal of $\mathcal{L}$, iff for any $K = M \oplus u \in \mathfrak{K}$ and any $X = H \oplus x \in \mathcal{L}$ we have $[X,K] = [H,M] + L(x,u) \oplus Hu - Mx \in \mathfrak{K}$. This is equivalent to

(i) $[H,M]$, $L(x,u) \in \mathfrak{M}$ for all $H \in \mathfrak{H}$, $x \in \mathfrak{T}$, $M \in \mathfrak{M}$, $u \in \mathfrak{U}$

(ii) Hu, $Mx \in \mathfrak{U}$ for all $H \in \mathfrak{H}$, $x \in \mathfrak{T}$, $M \in \mathfrak{M}$, $u \in \mathfrak{U}$

We define $i(\mathfrak{U}) = L(\mathfrak{T}, \mathfrak{U})$

$$j(\mathfrak{U}) = \{A \in \mathfrak{H}, A\mathfrak{T} \subset \mathfrak{U}\}$$

and get immediately from the above considerations,

<u>Lemma 4.</u> <u>$\mathfrak{K} \subset \mathcal{L}$ is a Θ-invariant ideal of $\mathcal{L}$, iff $\mathfrak{K} = \mathfrak{M} \oplus \mathfrak{U}$, where $\mathfrak{U}$ is an ideal of $\mathfrak{T}$, $\mathfrak{M}$ an ideal of $\mathfrak{H}$ such that $i(\mathfrak{U}) \subset \mathfrak{M} \subset j(\mathfrak{U})$.</u>

<u>Corollary 1.</u> $\mathfrak{J}(\mathfrak{U}) = i(\mathfrak{U}) \oplus \mathfrak{U}$. $\mathcal{J}(\mathfrak{U}) = j(\mathfrak{U}) \oplus \mathfrak{U}$ <u>are (Θ-invariant) ideals of $\mathcal{L}$ iff $\mathfrak{U}$ is an ideal of $\mathfrak{T}$.</u>