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Genetic algebras and time continuous models.
Theoret. Population Biology **4** (1973), 133–144.

Consider an algebra $\mathcal{A}$ which is genetic in R. D. Schafer's sense [Amer. J. Math. **71** (1949), 121–135; MR0027751 (10,350a)] or equivalently in H. Gonshor's [Proc. Edinburgh Math. Soc. (2) **17** (1970/71), 289–298; MR0302218 (46 #1371)] with weight function w. In what might now be called the classical view of genetic algebras, one considers in $\mathcal{A}$ a sequence of elements $G^{[n]} = \varphi^{n-1}G^{[1]}$ $(n = 1, 2, \cdots)$ generated from an arbitrary element $G^{[1]}$ by iteration of some operator $\varphi(\mathcal{A} \to \mathcal{A})$ such that $w(G^{[s]}) = 1$ whenever $w(G^{[1]}) = 1$. For example, $\varphi(G) = G^2$ determines the sequence of plenary powers. Then the sequence $G^{[n]}$ is called a train if for all $G^{[1]}$ of weight 1 it satisfies the same linear recurrence equation with constant coefficients.

Instead of such sequences $G^{[n]}$ the author considers elements $G(t) \in \mathcal{A}$ which are functions of the time t, one such function being determined by any starting value $G(0) = G_0 \in \mathcal{A}$, and such that $w(G(t)) = 1$ for all $t \geq 0$ whenever $w(G_0) = 1$. Then roughly speaking (the author puts it more carefully) this set of functions is said to form a continuous train if, for all G_0 of weight 1, $G(t)$ satisfies the same linear differential equation with constant coefficients.

With this apparatus genetic algebras are used to describe the variation in time of genotype frequencies in infinite populations under different mating systems. The cases considered include matings between individuals randomly drawn from the population at each moment, a population which is continuously backcrossed to a second constant population, and a population divided into two age groups which take part in the matings with different intensities. For the first case the general theory is applied to an example with tetraploids having a mixture of chromatid and chromosome segregation.

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