MR1224676 (95f:92018) 92D25 92-01 92D10
Lyubich, Yuri I. [**Lyubich, Yuriĭ Il′ich**] (1-SUNYS)

★**Mathematical structures in population genetics.** **(English summary)**

Translated from the 1983 Russian original by D. Vulis and A. Karpov.
Biomathematics, 22.
Springer-Verlag, Berlin, 1992. x+373 *pp.* $139.00. *ISBN* 3-540-53337-0

{The Russian original has been reviewed ["Naukova Dumka", Kiev, 1983; MR0731369 (85i:92001)].}

From the preface: "Mathematical methods have been applied successfully to population genetics for a long time. Even the quite elementary ideas used initially proved amazingly effective. For example, the famous Hardy-Weinberg law (1908) is basic to many calculations in population genetics. The mathematics in the classical works of Fisher, Haldane and Wright was also not very complicated but was of great help for the theoretical understanding of evolutionary processes. More recently, the methods of mathematical genetics have become more sophisticated. In use are probability theory, stochastic processes, nonlinear differential and difference equations and nonassociative algebras. First contacts with topology have been established. Now in addition to the traditional movement of mathematics for genetics, inspiration is flowing in the opposite direction, yielding mathematics from genetics. The present monograph reflects to some degree both patterns but especially the latter one. A pioneer of this synthesis was S. N. Bernstein. He raised—and partially solved—the problem of characterizing all stationary evolutionary operators, and this work was continued by the author in a series of papers (1971–1979). This problem has not been completely solved, but it appears that only certain operators devoid of any biological significance remain to be addressed. The results of these studies appear in Chapters 4 and 5. The necessary algebraic preliminaries are described in Chapter 3 after some elementary models in Chapter 2. The later chapters concern the dynamics of populations (Chapters 6, 7 and 9) and their formal analogue (Chapter 8). Here when selection is absent a very effective algebraic approach was introduced by Reiersöl (1961). This approach was extended by the author to describe explicit solutions of the general evolution equation (1971). These results are in Chapter 6 and some more abstract algebraic-dynamical theory, discovered by Etherington (1939) and his followers, is described in Chapter 7.

"The dynamics of selection (Chapter 9) require different methods. Here the leading role goes to Fisher's 'fundamental theorem' concerning the increase of mean fitness of a population due to natural selection. An approach based on this theory uses a relatively complicated 'relaxation' technique to establish the global convergence to equilibrium under conditions more general than have been previously achieved. Due to limitations of length we have omitted several topics which could be studied using these methods: polyploidy, overlapping generations, migration etc. Also not discussed in the book are stochastic considerations such as the effect of finite population size. These important topics deserve a separate presentation.

"References and comments are collected in a special appendix. We assume as background the standard material of linear algebra, mathematical analysis and probability theory."