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Heuch, I.
Sequences in genetic algebras for overlapping generations.
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I. M. H. Etherington introduced algebraic methods into the study of population genetics and pointed out that various sequences of elements in certain algebras correspond to different mating systems. (For example, the distribution of genotypes in successive generations in the simplest case can be conveniently expressed in terms of the sequence (a_n), where $a_{n+1}=(a_n)^2$, and the fundamental Hardy-Weinberg law on stability can be stated in the form $a_{n+1}=a_n$ if $n\geq 2$.) Subsequently, the concept of genetic algebra was introduced. This is an algebra over a field that possesses a basis $C_0,C_1,\cdots,C_n$ with multiplication given by $C_iC_j=\sum_k\gamma_{ijk}C_k$, where $\gamma_{000}=1$, γ_{0jk} for $k>j$, and γ_{ijk} for $k\leq\max(i,j)$ if $i>0$ and $j>0$. The weight of an element is the coefficient of C_0.

It turns out that many systems arising in genetics correspond to genetic algebras (in such a way that the set of genotype distributions corresponds to a subset of the elements of weight 1). The special nature of the multiplication table has algebraic consequences, some of which have direct genetic significance. Incidentally, the reviewer has shown that the above definition is equivalent to a basis-free definition of Schafer and has also given other equivalent basis-free definitions in a paper to appear.

In this paper the author studies a general class of sequences in genetic algebras generalizing work of Etherington [Proc. Roy. Soc. Edinburgh **59** (1939), 153–162; Jbuch **65**, 1138; ibid. **59** (1939), 242–258; MR0000597 (1,99e); J. London Math. Soc. **15** (1940), 136–149; MR0002854 (2,121h)], O. Reiersøl [Math. Scand. **10** (1962), 25–44; MR0137740 (25 #1189)] and P. Holgate [Proc. Edinburgh Math. Soc. (2) **15** (1966), 1–9; MR0201199 (34 #1083); J. London Math. Soc. **42** (1967), 489–496; MR0218413 (36 #1499); Proc. London Math. Soc. (3) **18** (1968), 315–327; MR0228269 (37 #3852)]. The sequences considered correspond to mating systems in which the genotype distribution in the nth generation depends not only upon distributions in the $(n-1)$th generation but also on previous generations. More specifically, it is assumed that a fixed proposition $2b_{hk}$ of the crosses to produce generation n is made between individuals from generations $n-h$ and $n-k$ and that $b_{hk}=0$ when $h,k\geq p$ for some fixed p. Thus we have a reasonable model of a system that allows for overlapping of generations and that takes fertility and assortative mating with respect to generation into account. The main result is that such a sequence always satisfies a difference equation which is the same for all initial elements of weight 1. The result as stated is much more detailed, giving information about the roots of the difference equation and upper bounds on their multiplicity. In the proof a difference equation is obtained inductively with respect to j for the sequence of coefficients of a fixed basis element C_j, using the tractable nature of the multiplication table of a genetic algebra. The difference equations for the various j's are then pieced together to get the difference equation for the sequence of elements. *H. Gonshor*