Lie triple systems

199A Problems (related to Lie triple systems) and Lie algebras

#1 $\delta(x) = \delta_{a,b}[x] = [[a,b],x]$ $\quad a, b, x \in M_n(\mathbb{C})$

$\underbrace{a,b}_{\text{fixed}} \quad \underbrace{x}_{\text{variable}} \in M_n(\mathbb{C})$

$\Rightarrow \quad \delta[x,y,z] = [\delta x, y, z] + [x, \delta y, z] + [x, y, \delta z]$

where $[a,b,c] = [[a,b],c]$

$$= (ab - ba)c - c(ab - ba)$$

#3 $M_n(\mathbb{R})$ is a Lie triple system under triple bracket multiplication $[a,b,c] = [[a,b],c]$

i.e.

- $[a,a,b] = 0$

- $[a,b,c] + [b,c,a] + [c,a,b] = 0$

- $[d,e,[a,b,c]] = [[d,e,a],b,c] + [a,[d,e,b],c]$
$$+ [a,b,[d,e,c]]$$

#8 A Lie algebra, with product $[a,b]$

is a Lie triple system under $[a,b,c] = [[a,b],c]$

Assumption $[a,a] = 0$ & $[[a,b],c] + [[b,c],a] + [[c,a],b] = 0$

conclusion The three bullets in #3