Elementary Analysis Math 140B—Winter 2007
Homework answers—Assignment 13; February 26, 2007

Exercise 28.10, page 212

Let $h(x) = [\cos x + e^x]^{12}$

(a) Calculate $h'(x)$.

Solution: $h'(x) = 12[\cos x + e^x]^{11}(-\sin x + e^x)$

(b) Show how the chain rule justifies your computation in part (a) by writing $h = g \circ f$ for suitable f and g.

Solution: $g(x) = x^{12}$, $f(x) = \cos x + e^x$.

Exercise 28.12, page 213

(a) Differentiate the function whose value at x is $\cos(e^{x^5-3x})$.

Solution: $-\sin(e^{x^5-3x})(e^{x^5-3x})(5x^4 - 3)$.

(b) Use Exercise 28.11 or Theorem 28.4 to justify your computation in part (a).

Solution: Let $h(x) = \cos x$, $g(x) = e^x$, $f(x) = x^5 - 3x$. By Exercise 28.11,

$$(h \circ g \circ f)'(x) = h'(g \circ f(x))g'(f(x))f'(x).$$

Alternatively, by Theorem 28.4, writing $h \circ g \circ f = h \circ (g \circ f)$ we have $(g \circ f)'(x) = g'(f(x))f'(x)$ and

$$(h \circ g \circ f)'(x) = [h \circ (g \circ f)]'(x) = h'(g \circ f(x))(g \circ f)'(x) = h'(g \circ f(x))g'(f(x))f'(x).$$

Exercise 28.14(b), page 213

Suppose that f is differentiable at a. Prove that

$$\lim_{h\to 0} \frac{f(a+h) - f(a-h)}{2h} = f'(a).$$

Solution:

$$\frac{f(a+h) - f(a-h)}{2h} = \frac{f(a+h) - f(a)}{2h} + \frac{f(a-h) - f(a)}{-2h} \to f'(a)/2 + f'(a)/2.$$