quiz 5 - 5:00PM

① $y'' - ty' + y = 0 \qquad (1)$

Ansatz: $y(t) = \sum_{n=0}^{\infty} a_n t^n$

$$\Rightarrow \quad y'(t) = \sum_{n=0}^{\infty} a_n n t^{n-1}$$

$$y''(t) = \sum_{n=0}^{\infty} a_n n(n-1) t^{n-2}$$

$$\overset{(1)}{\Rightarrow} \sum_{n=0}^{\infty} a_n n(n-1) t^{n-2} - t\sum_{n=0}^{\infty} a_n n t^{n-1} + \sum_{n=0}^{\infty} a_n t^n =$$

$$= \sum_{n=0}^{\infty} a_n n(n-1) t^{n-2} - \sum_{n=0}^{\infty} a_n n t^n + \sum_{n=0}^{\infty} a_n t^n =$$

$$= \sum_{n=-2}^{\infty} a_{n+2} (n+2)(n+1) t^n - \sum_{n=0}^{\infty} a_n n t^n + \sum_{n=0}^{\infty} a_n t^n =$$

$$= \sum_{n=0}^{\infty} \{ a_{n+2} (n+2)(n+1) - (n-1) a_n \} t^n = 0$$

$$\Rightarrow \quad a_{n+2}(n+2)(n+1) - (n-1)a_n = 0$$

$$\Rightarrow \quad a_{n+2} = \frac{n-1}{(n+2)(n+1)} a_n$$

one linearly indep. sln.:

$a_0 = 1 \quad a_1 = 0$:

$$\left.\begin{aligned} a_2 &= \frac{0-1}{2\cdot 1} = -\frac{1}{2} \\ a_4 &= \frac{2-1}{4\cdot 3}\left(-\frac{1}{2}\right) = \frac{-1}{2\cdot 3\cdot 4} \end{aligned}\right\} \Rightarrow y(t) = a_0\left(1 - \frac{1}{2}t^2 + \right.$$

$$\left. - \frac{1}{2\cdot 3\cdot 4} t^4 + \ldots \right)$$

$$y'' + y' + y = e^t, \quad y(0) = 2, \quad y'(0) = 0.$$

<u>denote:</u> $Y(s) := (\mathcal{L}y)(s)$

$$\Rightarrow \left(s^2 Y(s) - s \cdot \underbrace{y(0)}_{=2} - \underbrace{y'(0)}_{=0}\right) + s \cdot Y(s) - \underbrace{y(0)}_{=2} + Y(s) = \frac{1}{1-s}$$

$$\Rightarrow (s^2 + s + 1) Y(s) - 2(s+1) = \frac{1}{1-s}$$

$$\Rightarrow \quad Y(s) = \frac{1}{(1-s)(s^2+s+1)} + \frac{2(s+1)}{s^2+s+1}$$