(iii) Want to see whether a sln exists on $[-2,2]$:
Since f has properties of thm. (i) <u>on all of $\mathbb{R}^2$</u>, there exist a sln of (1) on $\mathbb{R}$.

(iv) $x=2$

$$x(t) = \underbrace{x(0)}_{=1} + \int_0^t x(s)\, s\, ds$$

$$x^0(t) = x(0) = 1$$

$$x^{(1)}(t) = 1 + \int_0^t s\, ds = 1 + \frac{t^2}{2}$$

$$x^{(2)}(t) = 1 + \int_0^t x^{(1)}(s)\, s\, ds = 1 + \int_0^t \left(1 + \frac{s^2}{2}\right) s\, ds =$$

$$= 1 + \frac{t^2}{2} + \frac{t^4}{2\cdot 4}$$

$$x^{(3)}(t) = 1 + \int_0^t x^{(2)}(s)\, s\, ds = 1 + \frac{t^2}{2} + \frac{t^4}{2\cdot 4} + \frac{t^6}{2\cdot 4\cdot 6}$$

$\vdots$

$$x^{(n)}(t) = 1 + \frac{t^2}{2} + \frac{t^4}{2\cdot 4} + \dots + \frac{t^{2n}}{2\cdot 4 \cdot \ldots \cdot 2n}$$

$$= 1 + \left(\frac{t^2}{2}\right) + \frac{1}{2}\left(\frac{t^2}{2}\right)^2 + \dots + \frac{1}{n!}\left(\frac{t^2}{2}\right)^n$$

$$= \sum_{k=0}^{n} \frac{1}{k!}\left(\frac{t^2}{2}\right)^k \xrightarrow{n\to\infty} \underline{\underline{e^{t^2/2}}}$$

... sln. to (1).

2.) $y'' + 4y' + 3y = 0$

characteristic eqn: $\lambda^2 + 4\lambda + 3 = 0$

$$\lambda_{1,2} = -\frac{4}{2} \pm \sqrt{\frac{16}{4} - 3} =$$

$$= -2 \pm \sqrt{\underbrace{4-3}_{=1}} = \underline{\underline{-1, -3}}$$

$\Rightarrow$ <u>gen. sln.:</u> $y(t) = C_1 e^{-t} + C_2 e^{-3t}, \quad C_1, C_2 \in \mathbb{R}$.