(iii) Want to see whether a sln exists on $[-4,4]$:

Since f has properties of thm $\overset{(1)}{\uparrow}$ on entire $\mathbb{R}^2$
$\Rightarrow \exists$ soln. to (1) on $\mathbb{R}$.

; choose a, b in thm s.t.

$$\alpha = 4$$

$$M = \max_{(t,x)\in \underbrace{[-a,a]\times[-b,b]}_{=R}} |f(t,x)| = \max_{(t,x)\in R} 3|x||t^2| = 3ab^2$$

$$\Rightarrow \quad \alpha = \min\{a, \frac{b}{M}\} = \min\{a, \frac{\cancel{b}}{3ab^{\cancel{2}}}\} =$$

$$= \min\{a, \frac{1}{3ab}\}$$

choose $a = 4$ and

b s.t. $\quad \dfrac{1}{3ab} = \dfrac{1}{12b} = 4$

$$\Leftrightarrow \quad b = \frac{1}{12\cdot 4}.$$

$\Rightarrow \alpha = 4 \Rightarrow$ solution $\overset{\text{of (1)}}{\uparrow}$ exits on

$$t \in [-4,4].$$

(iv) $$x(t) = x(0) + \int_0^t f(s, x(s))\,ds =$$

$$= 1 + \int_0^t 3x(s)s^2\,ds$$

$$x^{(0)} = 1$$

$$x^{(1)} = 1 + \int_0^t 3\underbrace{x^{(0)}(s)}_{=1}s^2\,ds = 1 + t^3$$

$$x^{(2)} = 1 + \int_0^t 3x^{(1)}(s)s^2\,ds = 1 + \int_0^t (3s^3+3)s^2\,ds =$$

$$= 1 + 3\int_0^t s^5\,ds + 3\int_0^t s^2\,ds =$$

$$= 1 + t^6\frac{1}{2} + t^3$$