solutions to quiz 3 : 9AM

(1.)

(i) thm.: Consider the initial value problem (IVP)

$$\begin{cases} x'(t) = f(t, x(t)) , \\ x(t_0) = x_0 \end{cases} \qquad (*)$$

Let $f : [t_0 - a, t_0 + a] \times [x_0 - b, x_0 + b] \longrightarrow \mathbb{R}$ continuous and $\frac{\partial f}{\partial x}$ exists on the domain of f and is continous, then the given IVP has a unique solution, i.e. $\exists\ \phi \in C'[t_0 - \alpha, t_0 + \alpha]$, $\phi(t_0) = x_0$ s.t. ϕ sattisfies $(*)$. Here,

$$\alpha := \min\left\{a, \frac{b}{M}\right\}, \text{ where}$$

$$M = \max_{(t,x)\in R} |f(t,x)|$$

$$R := [t_0 - a, t_0 + a] \times [x_0 - b, x_0 + b].$$

(ii) Check whether (1) sattisfies requirements in the thm. (i)

$$f(t,x) := 3xt^2$$

$\Rightarrow$ f is continuous on $\mathbb{R}^2$

$\frac{\partial f}{\partial x} = 3t^2$ is continous on $\mathbb{R}^2$

$\Rightarrow$ (1) has a unique sln. according to the thm. of Picard-Lindelöff.