

$$(2) \quad 2x + 3x^2y + (x^3 - 3y^2)y' = 0$$

check exactness:

$$\left. \begin{aligned} \frac{\partial}{\partial y} (2x + 3x^2y) &= 3x^2 \\ \frac{\partial}{\partial x} (x^3 - 3y^2) &= 3x^2 \end{aligned} \right\} =$$

\Rightarrow exact.

$$\Rightarrow \exists \phi \in C^2(\mathbb{R}^2) \quad \text{s.t.} \quad \frac{\partial \phi}{\partial x} = 2x + 3x^2y \quad (i)$$

$$\text{and} \quad \frac{\partial \phi}{\partial y} = x^3 - 3y^2 \quad (ii)$$

$$(i) \Leftrightarrow \phi = \int (2x + 3x^2y) dx + h(y), \quad \text{some } h \in C^1 \\ = x^2 + x^3y + h(y)$$

$$(ii) \Rightarrow x^3 - 3y^2 = \frac{\partial \phi}{\partial y} = x^3 + h'(y)$$

$$\Rightarrow h'(y) = -3y^2$$

$$\Rightarrow h(y) = -y^3 + \tilde{C}, \quad \tilde{C} \in \mathbb{R}.$$

Thus,

$$\phi = x^2 + x^3y - y^3 + \tilde{C}$$

and therefore:

general sol. of (2):

$$\underline{x^2 + x^3y - y^3 = C, \quad C \in \mathbb{R}.$$