

Solutions - quiz 2

5:00 PM

$$\begin{aligned} \textcircled{1} \quad (yt + 3t^2)y' &= y^2 + 2yt - 3t^2 \\ y' &= \frac{y^2 + 2yt - 3t^2}{yt + 3t^2} = \frac{y^2 + 2yt - 3t^2}{t^2} \cdot \frac{t^2}{yt + 3t^2} = \\ &= \frac{\left(\frac{y}{t}\right)^2 + 2\left(\frac{y}{t}\right) - 3}{\left(\frac{y}{t}\right) + 3} \quad \textcircled{1a} \end{aligned}$$

\Rightarrow right hand side is of the form $f\left(\frac{y}{t}\right)$ ("homogeneous deg.")

change of variable: $\eta := \frac{y}{t}$

$$\begin{aligned} \Rightarrow \eta'(t) &= \frac{y'(t)}{t} - \frac{1}{t^2}y(t) = \frac{y'(t)}{t} - \frac{1}{t}\eta(t) = \\ &= \frac{1}{t} [y'(t) - \eta(t)]. \end{aligned}$$

$$\Rightarrow y'(t) = t\eta'(t) + \eta(t)$$

$$\textcircled{1a} \Rightarrow t\eta'(t) + \eta(t) = \frac{\eta^2 + 2\eta - 3}{\eta + 3}$$

$$\text{since: } \eta^2 + 2\eta - 3 = (\eta + 3)(\eta - 1)$$

$$\Rightarrow t\eta' + \eta = \frac{(\eta + 3)(\eta - 1)}{\eta + 3} = \eta - 1$$

$$\Rightarrow \eta' = -\frac{1}{t} \Rightarrow \eta(t) = -\ln|t| + C, C \in \mathbb{R}.$$

Therefore, (substituting back): $y(t) = -t \ln|t| + Ct, C \in \mathbb{R}.$