

$$(2) \quad (yt + t^2)y' = y^2 - t^2$$

$$y' = \frac{y^2 - t^2}{yt + t^2} = \frac{\frac{y^2 - t^2}{t^2}}{\frac{yt + t^2}{t^2}} = \frac{\left(\frac{y}{t}\right)^2 - 1}{\left(\frac{y}{t}\right) + 1} \quad (2a)$$

$\Rightarrow$  right hand side is of the form  $f\left(\frac{y}{t}\right)$  ("homogeneous deg.")

change of variable:  $\eta(t) := \frac{y(t)}{t}$ .

$$\Rightarrow \eta'(t) = \frac{y'(t)}{t} - \frac{y(t)}{t^2} = \frac{1}{t} [y'(t) - \eta(t)]$$

$$\Rightarrow y'(t) = t\eta'(t) + \eta(t).$$

$$(2a) \Rightarrow t\eta'(t) + \eta(t) = \frac{\eta^2 - 1}{\eta + 1} = \frac{(\eta - 1)(\eta + 1)}{(\eta + 1)}$$

$$\Rightarrow t\eta' + \cancel{\eta} = \cancel{\eta} - 1$$

$$\Rightarrow \eta' = -\frac{1}{t}$$

$$\Rightarrow \eta(t) = -\ln|t| + C, \quad C \in \mathbb{R}.$$

Therefore, (substituting back):

$$\underline{\underline{y(t) = -t \ln|t| + Ct, \quad C \in \mathbb{R}.}}$$