Introduction to differential equations.

Midterm Review.

- Differential equations of the first order
  - Linear
  - Separable
  - Exact
  - Homogeneous
  - The existence and uniqueness theorem
- Differential equations of the second order
  - Linear homogeneous and non-homogeneous, properties of solutions, Wronskian
  - Linear homogeneous with constant coefficients.
  - Linear non-homogeneous, method of variation of parameters
  - Linear non-homogeneous, with constant coefficients and special r.h.s. (polynomial, exponential function, $\sin \omega t$, $\cos \omega t$, or product of those)
  - Reduction of order.

Midterm: 4 problems

2 - first order

2 - second order

One of the problems is „theoretical".

40 points + 1 (evaluation forms)

Problem 1

$$y' + a(t)y = f(t),$$

$a(t), f(t)$ are continuous for $t \in \mathbb{R}$,

$$a(t) \geq c > 0$$

$$f(t) \to 0 \text{ as } t \to +\infty$$

Prove that for every solution $y(t)$

$$y(t) \to 0 \text{ as } t \to +\infty.$$

Problem 2

Find a continuous solution of the IVP

$$y' + y = g(t), \quad y(0) = 0$$

$$g(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & t > 1 \end{cases}$$

Problem 3

Is it possible that the graphs of two different solutions of the equation

$$y' = t + y^2$$

intersects at some point (t_0, y_0)?

The same question for

$$y'' = t + y^2.$$

Problem 4

$$y'' + p(t)y' + q(t)y = 0$$

Assume that $q(t) < 0$. Prove that a solution $y(t)$ cannot have a positive maximum.

Problem 5

A linear non-homogeneous equation of second order have solutions $y_1 = 1$, $y_2 = t$, $y_3 = t^2$.
Find a general solution.

Problem 6

For which a, b
every solution of the equation

$$y'' + ay' + by = 0$$

has infinite number of zeros?

Problem 7

For which $k, \omega > 0$
the equation $y'' + k^2 y = \sin \omega t$
has at least one periodic solution?

Problem 8

Find a general solution

$$y'' = 2t\,y'$$

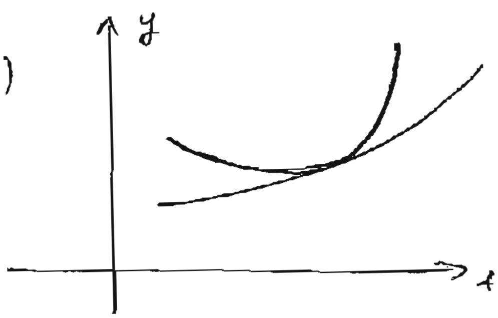
Problem 9

Is it possible that the graphs of two solutions of the differential equation

$$y'' + q(t)\,y = 0 \quad , \; q(t) \text{ is continuous,}$$

have the form:

a)



b)