MATH 3D, FALL 2008

Final exam (sample), v.1

Problem 1.

Find the general solution of the equation

$$ty'' + 2y' = 0$$

Problem 2.

Compute first three Picard iterations for the initial value problem

$$y' = t - y^2,\ y(0) = 0.$$

Problem 3.

Find the inverse Laplace transform of the function

$$F(s) = \frac{s}{s^2 + 4s + 8}$$

Problem 4.

The equation

$$y'' - 2ty' + \lambda y = 0,$$

where λ is a constant, is known as the Hermite differential equation. Find two linearly independent series solutions of the Hermite equation.

Problem 5.

Find the general solution of the system of equations

$$\frac{d\overline{x}}{dt} = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix} \overline{x}$$

Final exam (sample), v.2

Problem 1.

Find the general solution of the equation

$$\cos y \sin t \frac{dy}{dt} = \sin y \cos t$$

Problem 2.

Check that $y_1(t) = te^t$ and $y_2(t) = (t-2)e^t$ are solutions of the equation

$$ty'' - (t+1)y' + y = (t-1)e^t.$$

Find the general solution.

Problem 3.

Find the inverse Laplace transform of the function

$$F(s) = \frac{4}{(s-1)^2}$$

Problem 4.

Check that $y(t) = t^2$ is a solution of the equation

$$t^2 y'' - 4ty' + 6y = 0.$$

Find the general solution.

Problem 5.

Find the solution of the initial value problem

$$\frac{d\overline{x}}{dt} = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} \overline{x}, \quad \overline{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

Final exam (sample), v.3

Problem 1.

Find the general solution of the equation

$$ty^2\frac{dy}{dt} = t^2 + y^3$$

Problem 2.

Find the Wronskian of $y_1(t) = e^t \cos t$ and $y_2(t) = e^t \sin t$.

Problem 3.

Find the inverse Laplace transform of the function

$$F(s) = \frac{s}{(s-1)^2}$$

Problem 4.

Three solutions of a certain second-order linear nonhomogeneous equation are

$$\psi_1(t) = 1 + t,\ \psi_2(t) = (1+t^2)t,\ \ \psi_3(t) = t^3 + t + 1.$$

Find the general solution.

Problem 5.

Find the general solution of the system of equations

$$\frac{d\overline{x}}{dt} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \overline{x}$$