Math 3D - extra credit quiz

November 24, 2008

Instructions

Read these instruction carefully.

1. This is a set of problems, to earn extra credit. Participation is voluntary.

2. Participation can only improve your performance. It gives you a chance to improve the score you earned in the quizzes.

3. Upon participation, this set of problems counts as an additional quiz you completed. Should you participate, the quizzes with the *three* lowest scores will be dropped; this is one more than what would be dropped without participation. This means, should you do worse than your lowest quiz score so far, than the extra credit quiz will be the third quiz dropped. Should you do well, than you have the opportunity to exchange your lowest quiz score.

4. This is a take home quiz. It is due at on Wednesday, November 26, at the beginning of class (9:00 AM).

5. You can use whatever material (book, internet, etc.) you wish. You have to however do the problems ON YOUR OWN. Communication with other human beings about these problems is not permitted.

6. Show all your work and clearly define all the symbols you use.

Problems

1. Find the orthogonal curves for the family of functions, $y(t) = Ce^t$.

2. Consider the differential equation $y''(t) + p(t)y'(t) + q(t)y(t) = 0$, where $p(t)$ and $q(t)$ are continuous functions on the real line. Suppose that all its solutions $y(t)$ fulfill, $y(t) \to 0$ and $y'(t) \to 0$ as $t \to +\infty$. Prove that $\int_0^A p(t)\mathrm{d}t \to +\infty$ as $A \to +\infty$.

3. Let $y(t)$ be a solution of the differential equation $y''(t) + p(t)y'(t) + q(t)y(t) = 0$. Here, $p(t)$ and $q(t)$ are continuous functions on the real line. Suppose that this solution satisfies, $|y(t)| \leq t^2$, for all $t \in \mathbb{R}$. Find all such $y(t)$. (Hint: Given that $|y(t)| \leq t^2$, all real t, what can you conclude about $y(0)$ and $y'(0)$? Try to use the uniqueness theorem.)