INTRO DIFFERENTIAL EQUATIONS

Midterm Exam: Answers

Version 1.

Problem 1.
Find a general solution of the equation

$$(\sin t + y)\frac{dy}{dt} + y\cos t - t^2 = 0$$

Answer: $y\sin t - \frac{t^3}{3} + \frac{y^2}{2} = C$

Problem 2.
Find the solution of the initial value problem

$$t\frac{dy}{dt} + t^2 + ty - y = 0, \quad y(1) = 0.$$

Answer: $y(t) = t(e^{1-t} - 1)$

Problem 3.
Find a general solution of the equation

$$y'' - 2y' + y = \frac{e^t}{t}$$

Answer: $y(t) = e^t(t\ln|t| + C_1 t + C_2)$

Problem 4.
What second order linear homogeneous differential equation with constant coefficients has a solution $y(t) = e^{2t}\sin 4t$?

Answer: $y'' - 4y' + 20y = 0$

Version 2.

Find a general solution of the equation

$$y \sin t + y' \cos t = 1$$

Answer: $y(t) = C \cos t + \sin t$

Problem 2.
Find the solution of the initial value problem

$$y - y' = y^2 + ty', \quad y(1) = 0.$$

Answer: $y(t) \equiv 0$

Problem 3.
Find a general solution of the equation

$$y'' + 3y' + 2y = \frac{1}{e^t + 1}$$

Answer: $y(t) = (e^{-t} + e^{-2t}) \ln(e^t + 1) + C_1 e^{-t} + C_2 e^{-2t}$

Problem 4.
What second order linear homogeneous differential equation with constant coefficients has a solution $y(t) = 2e^t \sin t$?

Answer: $y'' - 2y' + 2y = 0$

Version 3.

Find a general solution of the equation

$$ty' = y - te^{\frac{y}{t}}$$

Answer: $y(t) = -t \ln \ln(Ct)$

Find the solution of the initial value problem

$$y = t(y' - t\cos t), \quad y(\pi) = \pi.$$

Answer: $y(t) = t(1 + \sin t)$

Find a general solution of the equation

$$y'' + y = \frac{1}{\sin t}$$

Answer: $y(t) = (C_1 + \ln|\sin t|)\sin t + (C_2 - t)\cos t$

What second order linear homogeneous differential equation with constant coefficients has a solution $y(t) = 5e^{-t}\cos t$?

Answer: $y'' + 2y' + 2y = 0$