INTRO DIFFERENTIAL EQUATIONS

Final Exam ₂

Wednesday, December 10, 2008 — 8:00 am - 10:00 am

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

Find a bounded solution of the differential equation

$$\frac{dy}{dt} = y + \cos t.$$

Problem 2.

Compute the first three Picard iterates ($y_0(t)$, $y_1(t)$, and $y_2(t)$) for the initial value problem

$$\frac{dy}{dt} = y^2 + 3t^2 - 1, \quad y(1) = 1.$$

Problem 3.

Solve the initial value problem

$$y'' + 3y' + 2y = 2 - H_1(t), \quad y(0) = 0, \ y'(0) = 0,$$

where

$$H_1(t) = \begin{cases} 0, & t < 1, \\ 1, & t \geq 1. \end{cases}$$

Problem 4.

Check that $y_1(t) = 2t$ and $y_2(t) = (t+1)^2$ are solutions of the equation

$$(3t^3 + t)y'' + 2y' - 6ty = 4 - 12t^2.$$

Find the general solution.

Problem 5.

Find the general solution of the system of equations

$$
\begin{cases}
\frac{dx_1}{dt} = 3x_1 - x_2 + x_3 \\
\frac{dx_2}{dt} = x_1 + x_2 + x_3 \\
\frac{dx_3}{dt} = 4x_1 - x_2 + 4x_3
\end{cases}
$$