Complex Analysis Math 220 C

Midterm Exam

Wednesday, May 26, 2010 — MSTB 114 — 12:00 pm - 1:00 pm

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

Let S be a sequence of points in the complex plane that converges to 0. Let $f(z)$ be defined and analytic on some disc centered at 0 except possibly at the points of S and at 0. Show that either $f(z)$ extends to be meromorphic in some disc containing 0, or else for any complex number w there is a sequence $\{\xi_j\}$ such that $\xi_j \to 0$ and $f(\xi_j) \to w$ as $j \to \infty$.

Problem 2.

Let $u(z)$ be a harmonic function on the complex plane $\mathbb{C}$ such that

$$\iint_{\mathbb{C}} |u(x+iy)|dxdy < \infty.$$

Prove that $u(z) \equiv 0$.

Problem 3.

Find an explicit conformal map of the open set $\mathbb{C}\backslash\{z \in \mathbb{R}, |z| \geq 1\}$ to the unit disc.

Problem 4.

Let $p(z) = z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0$ be a complex polynomial. Prove that there must be at least one point with $|z| = 1$ and $p(z) \geq 1$.

Problem 5.

Let $f(z) = u(x, y) + iv(x, y)$ be a non-constant analytic function on some open domain $U \subset \mathbb{C}$. Prove that the level curves $u(x, y) = constant$ and $v(x, y) = constant$ intersect at right angles.