COMPLEX ANALYSIS MATH 220 C

Midterm Exam

Monday, May 17, 2010 — MSTB 114 — 12:00 pm - 1:00 pm

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

Find the integral

$$\int_0^{+\infty} \frac{(\ln x)^2}{1 + x^2} dx$$

<u>Problem 2.</u>

Find explicitly a conformal mapping of the domain $\{0 < \operatorname{Im} z < 1\}$ to the domain $\{0 < \operatorname{Im} z < 1, \operatorname{Re} z > 0\}$.

Problem 3.

Show that each of the two series

$$\sum_{n=1}^{\infty} \frac{nz^n}{1-z^n} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{z^n}{(1-z^n)^2}$$

a) converges uniformly on compact subsets of the unit disc;

b) converges to *the same* holomorphic function.

Problem 4.

Let f be a function holomorphic in a neighborhood of the unit disc. Suppose that $|f(z) - z| < 1$ for every $|z| = 1$.

a) Prove that $|f'(1/2)| \leq 8$;

b) Prove that f has precisely one zero in the unit disc.

Problem 5.

Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be a power series with the radius of convergence $R = 2$. Determine the region of convergence of the Laurent series $\sum_{n=-\infty}^{\infty} a_{|n|} z^n$ and identify the function which it represents there.