

COMPLEX ANALYSIS MATH 220 C

Midterm Exam

Friday, May 7, 2010 — 12:00 pm - 1:00 pm

Problem	1	2	3	4	5	Σ
Points						

Student's name:

Problem 1.

Find the integral

$$\int_{-\infty}^{+\infty} \frac{\cos x}{e^x + e^{-x}}$$

Problem 2.

Show that the function

$$f(z) = \sum_{n=1}^{\infty} \frac{z^2}{n^2 z^2 + 1}$$

is meromorphic on \mathbb{C} . Determine the set of poles and their order.

Problem 3.

Find a function continuous on $\mathbb{H} \cup (\mathbb{R} \setminus \{0\})$ and harmonic on $\mathbb{H} = \{\operatorname{Im} z > 0\}$ that takes value 0 on positive real axis, and value 1 on negative real axis.

Problem 4.

Let f be analytic in a neighborhood of the unit disc D and assume $|f(z)| \leq 1$ for all $z \in \overline{D}$. Suppose also that $f(1/2) = f(i/2) = 0$. Prove that $|f(0)| \leq 1/4$.

Problem 5.

Let $\sum a_n z^n$ and $\sum b_n z^n$ be two power series, with radius of convergence r and s , respectively. What can you say about the radius of convergence of the series

a) $\sum (a_n + b_n) z^n$,

b) $\sum a_n b_n z^n$?